Laboratory Manual

for

Control System Lab

Prepared by

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A.C. POSITION CONTROL ACP-01

1. OBJECTIVE

To study the performance characteristics of an a.c. servo motor angular position control system, also referred to as a carrier control system.

2. EQUIPMENT DESCRIPTION

A major portion of any first course on automatic control system invariably revolves around the study of position control systems using either d.c or a.c. motors. Experimental work in this area has usually been confined to analog simulated systems, e.g. through our 'Linear System Simulator' or similar other units. Although the biggest advantage of this approach is the unlimited flexibility and near perfect operation of the simulated systems leading to a close correlation between theoretical and experimental results, the student is denied the feel of a physical electromechanical system. The present unit and also our ‘d.c. position control’ have been designed with the objective of working with a physical system. Despite the constraints like friction, dead zone, nonlinearities due to amplifier saturation and motor current limiting, and low speed of response associated with any mechanical system, the student has been provided with enough opportunity for experimentation on a working system. The schematic diagram in Fig. 1 shows a typical carrier control system using a pair of synchros as angular error detector and a 2-phase a.c. servomotor as the driver, driving the load through a gear train.

The present system, as shown in Fig. 2, uses a pair of servo potentiometers as error detector. Synchros are avoided in the unit due to their high cost although the behaviour of the overall system is independent of the type of error detector used. Synchros may however be studied in our experiments ‘Synchro Error Detector’ separately.

2.1 Signal Sources

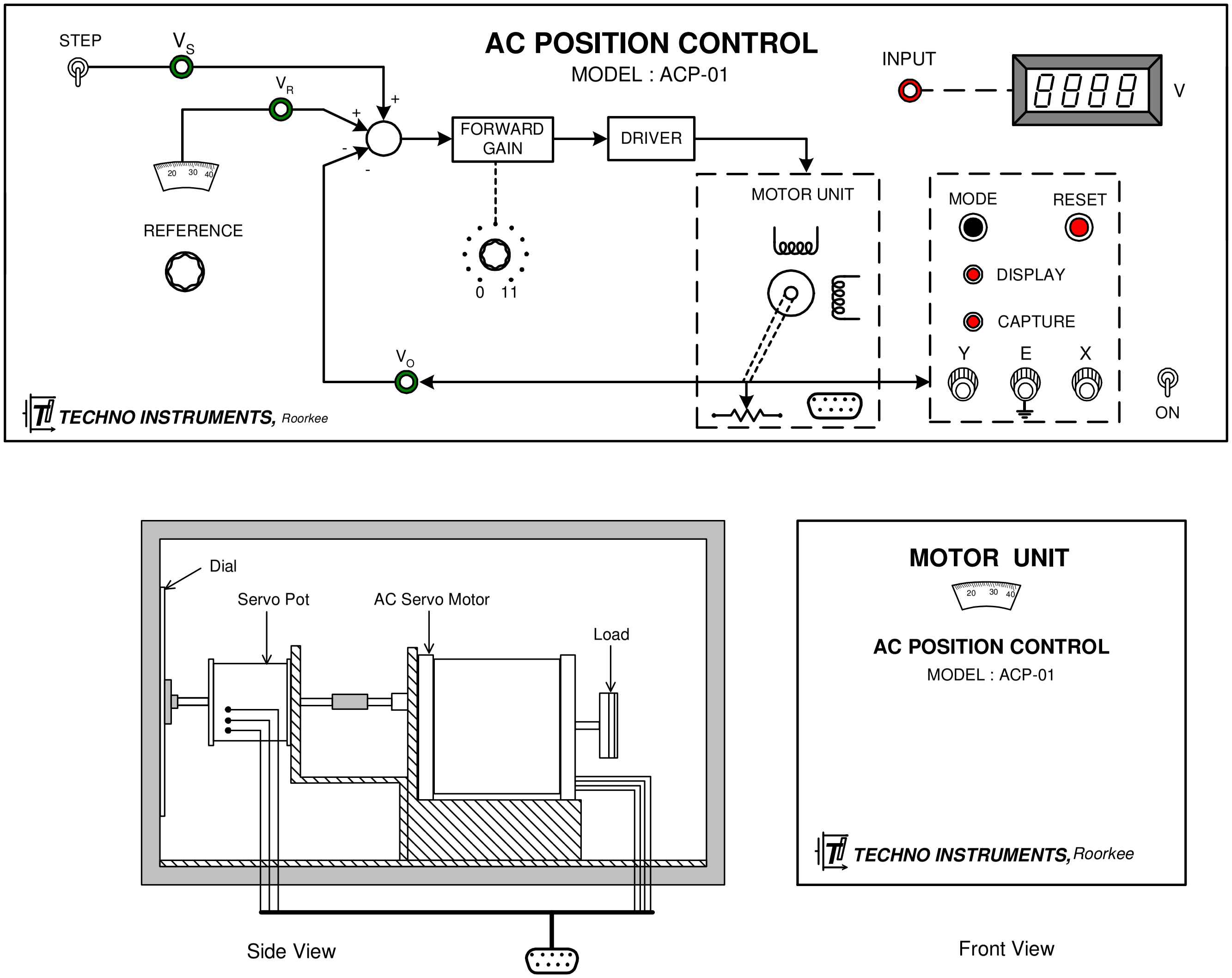
- Angle command (continuous): obtained through a potentiometer with a calibrated dial attached. The potentiometer is supplied with a 5 volt, 50 hz source.
- Angle command (step): available through a toggle switch. Automatic synchronisation with waveform capture circuit is provided so that the step response is stored properly.

2.2 Motor Unit

The position control is achieved through a good quality 2-phase a.c. gear motor. The specifications of the motor are :

- Operating voltage : 12V rms 50 Hz per phase.
- Phase current : 1.2 amp. (approx)
- Rated speed : 90 rpm
- Torque (basic) : 750 gm-cm (approx.)

Angular position of the motor shaft is sensed by an identical 360° rotation potentiometer attached to it, which is connected to the same 5V, 50 Hz source as that used in the command potentiometer. A calibrated disk mounted on the potentiometer indicates its angular position in degrees.



Panel diagram of A.C. Position Control, Model ACP-01

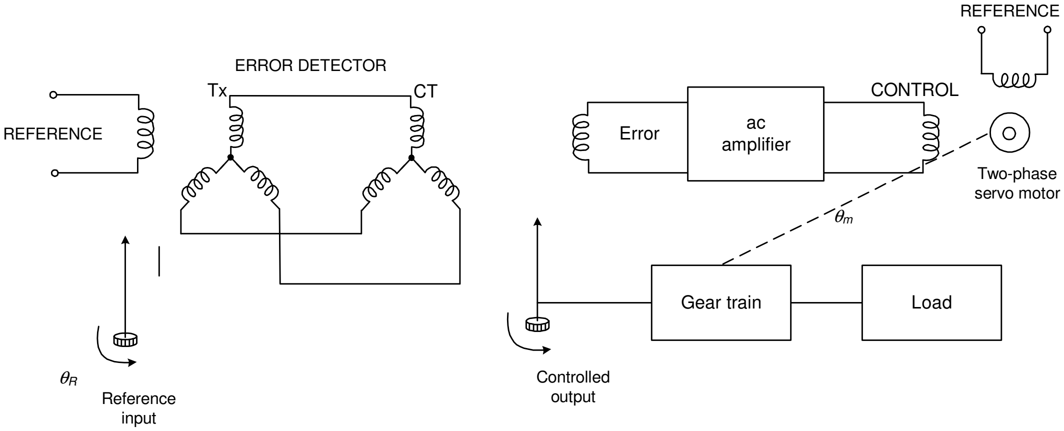


Fig. 1 Carrier control system employing synchro error detector

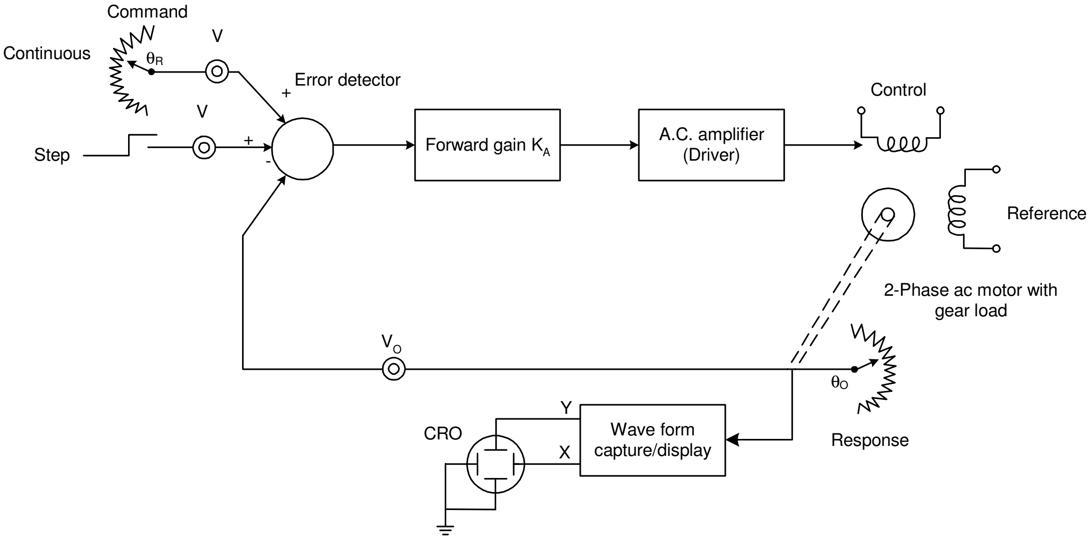


Fig. 2 System schematic

All the above components, viz. the motor, potentiometer, load etc. are fitted inside the 'motor unit'. Transparent panels provide a good view of the interior. The motor unit is connected to the rest of the system through a 9-pin D-type connector and cables.

2.3 Main Unit

The main unit houses the command circuit, the error detector and the gain control of the forward path, the power stage and the waveform capture/display unit. Different experiments are performed by appropriate settings of the controls as explained later. Description of the above blocks is given next.

(a) Command: Two operating modes have been provided in the system. When a continuous command is given by the rotation of a potentiometer through a certain angle, the closed loop system responds by an identical rotation of the motor shaft. Alternatively, a step command equivalent to about 150 degrees may be given by a switch. This is used for CRO studies of the step response.

(b) Error detector: This is a 3-input 1-output block. Two of the inputs are meant for command signals and the remaining input, having 180° phase shift, is used for position signal feedback.

(c) Gain blocks: The forward path gain is adjustable from 1 to 11 and may be read from the markings on the panel. This gain range is adequate to study both over damped and under damped response.

(d) Driver: The driver is a unity gain power amplifier suitable for running the motor upto full power in either direction. The voltage gain of this block is unity.

(e) Waveform Capture/Display unit: The time response of a mechanical system like the present one is usually too slow for a CRO display, except on a storage oscilloscope. Alternatively an X-Y recorder could be used to get a hard copy which may subsequently be studied quantitatively. Both these options are quite expensive for an usual undergraduate laboratory. The waveform capture/display unit is a microprocessor based card which can 'capture' the motor response and then 'display' the same on any ordinary X-Y oscilloscope for a detailed study. The stored waveform is erased whenever another waveform is captured, or the unit is reset.

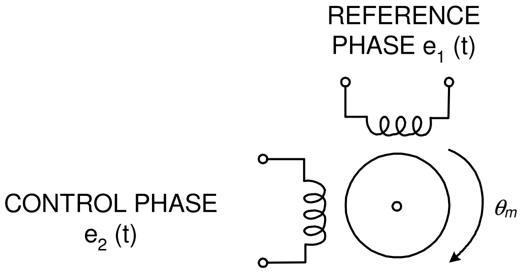
2.4 Power Supply

The set-up has a number of IC regulated supplies which are permanently connected to all the circuits. **No external supply should be connected to the terminals on the panel**.

Capabilities of this unit include an evaluation of the performance of the position control system for different values of forward gains. Effect of non-linearity, so common in all practical systems, may be readily observed by the student. In all the cases the response is stored and can then be displayed on an ordinary measuring oscilloscope.

3. BACKGROUND SUMMARY

Second order systems are studies in great detail in any course on linear control system. The reason for this is that a large number of higher order practical control systems may be approximated as a second order system while neglecting less dominant modes, nonlinearities like dead zone, saturation, hysteresis etc., assuming these to have little effect on the performance. Also second order systems lend themselves to simple and accurate mathematical analysis. In the following description we shall follow the above strategy. At the end however, the imperfections due to nonlinearities shall be pointed out.



$$G(s)=\frac{\theta_m(s)}{E_2(s)} \quad e_1(t) \text{ fixed}$$

Fig. 3 Transfer function of a two phase servo motor

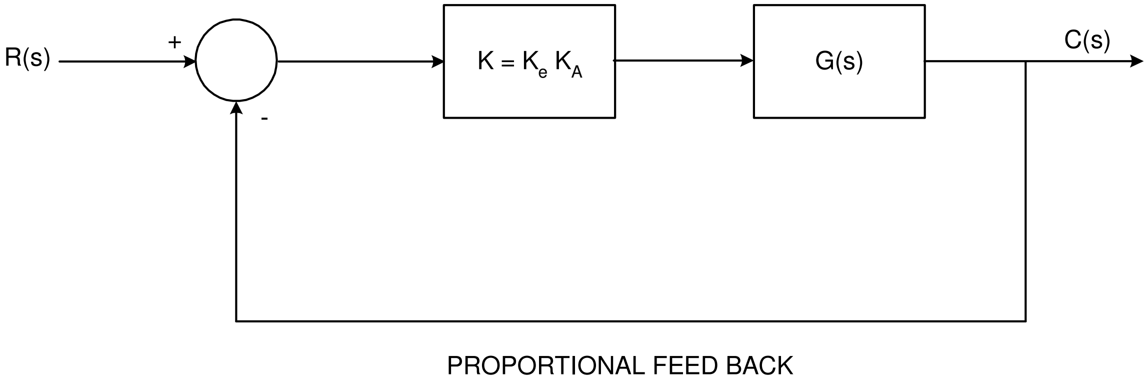


Fig. 4 Simplified block diagram

3.1 Two Phase Servomotor

Referring to Fig. 3 transfer function of a 2-phase servomotor may be derived as [3]

$$G_m(s) = \frac{\theta_m(s)}{E_2(s)} = \frac{K_m}{s(s\tau_m + 1)}$$

Where, K_m is the motor gain constant, and

τ_m is the motor time constant

In view of this, a simplified block diagram of the position control system may be drawn as shown in Fig. 4. Notice that the forward gain K in this diagram includes the sensitivity of the error detector, K_e, as well , viz.,

$$K_e = \frac{\text{error signal in volts}}{\text{angular error in radians}}$$

Thus $G(s) = KG_m(s)$

The overall transfer function of the position control systems is then given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{KK_m}{(s^2\tau_m + s + KK_m)},$$

which may be expressed in the standard form by dividing numerator and denominator by τ_m and redefining the notations as shown in the next section.

3.2 Position Control - a second order stem

A second order system is represented in the standard form as,

$$G(s) = \frac{\omega_n{}^2}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

where ζ is called the damping ratio and ω_n the undamped natural frequency. Depending upon the value of ζ, the poles of the system may be real, repeated or complex conjugate which is reflected in the nature of its step response. Results obtained for various cases are :

(a) Under damped case $(0 < \zeta < 1)$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{(1-\zeta^2)}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{(1-\zeta^2)}}{\zeta}\right) \qquad (1)$$

where, $\omega_d = \omega_n\sqrt{(1-\zeta^2)}$ is termed the damped natural frequency. A sketch of the unit step response for various values of ζ is available in the text books.

(b) Critically damped case $(\zeta = 1)$

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \qquad (2)$$

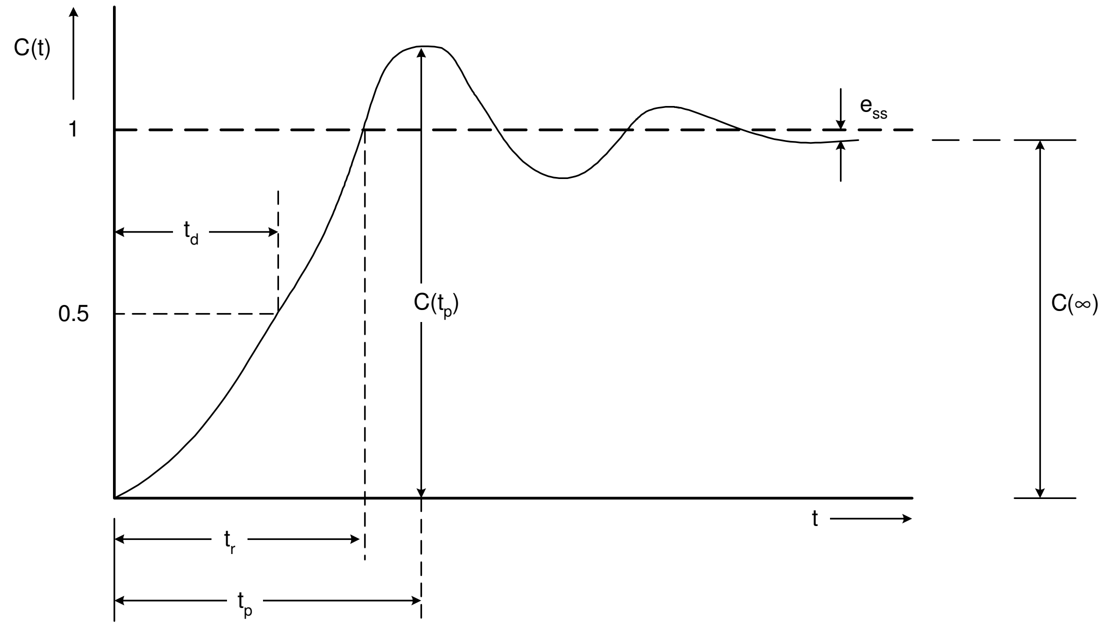


Fig. 5 (a) Unit step response of a normalized second order transfer function (Under Damped)

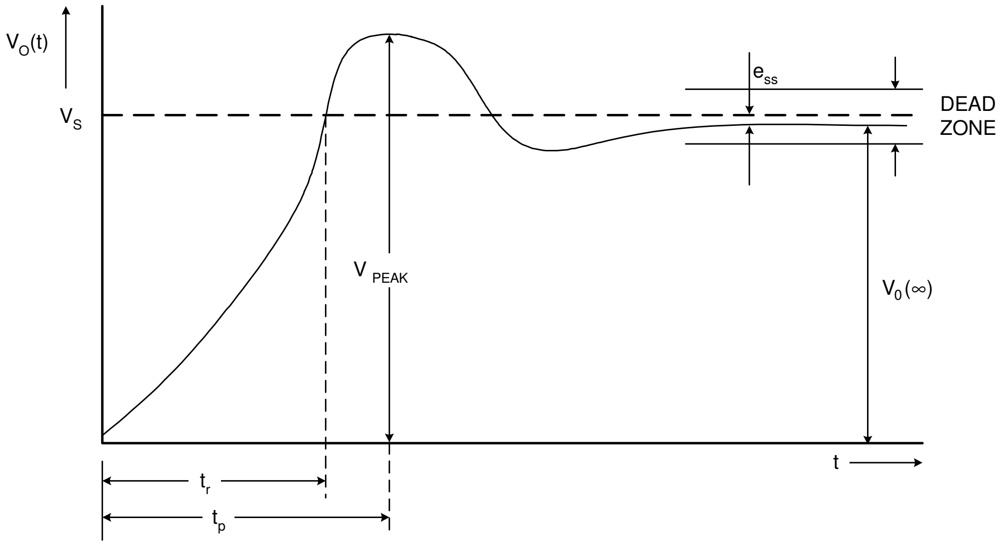


Fig. 5 (b) Typical step response of the position control system

(c) Over damped case ($\zeta > 1$)

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{(1-\zeta^2)}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right) \qquad (3)$$

where $s_1 = (\zeta + \sqrt{(\zeta^2 - 1)}\,)\omega_n$ and $s_2 = (\zeta - \sqrt{(\zeta^2 - 1)}\,)\omega_n$

Unit step response similar to equations (1), (2) or (3), depending upon the value of K_A may thus be expected in the position control system. Thus the response of the position control system can be altered by varying the amplifier gain K_A, and a 'satisfactory' performance may usually be obtained. This leads to the concept of performance characteristics as defined on the step response of an underdamped second order system in Fig. 5(a) and explained in brief here.

(i) ***Delay time,*** t_d, is defined as the time needed for the response to reach 50% of the final value.

(ii) ***Rise time,*** t_r, is the time taken for the response to reach 100% of the final value for the first time. This is given by

$$t_r = \frac{\pi - \beta}{\omega_d}, \text{ where } \beta = \tan^{-1}\frac{\sqrt{(1-\zeta^2)}}{\zeta}$$

(iii) ***Peak time,*** t_p, is the time taken for the response to reach the first peak of the overshoot and is given by

$$t_p = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$$

(iv) ***Maximum overshoot,*** M_p, is defined by

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

(v) ***Settling time***, t_s, is the time required by the system response to reach and stay within a prescribed tolerance band which is usually taken as ±2% or ±5%. An approximate calculation based on the envelops of the response for a low damping ratio system yields

t_s (±5% tolerance band) = $3/\zeta\omega_n$

t_s (±2% tolerance band) = $4/\zeta\omega_n$

(vi) ***Steady State Error*** Another important characteristic of a closed loop system is the steady state error, e_{ss}. For unity feedback systems e_{ss} is defined as

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty}\{r(t) - c(t)\}$$

A simpler way to calculate steady state error without actually computing the time response is available in the complex frequency domain. Application of the final value theorem of Laplace Transform to unity feedback system gives,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} = 0,$$

$$\text{for } R(s) = \frac{1}{s} \text{ and } G(s) = \frac{KK_m}{s(s\tau_m + 1)}$$

Steady state error may be obtained for various inputs (step, ramp, parabolic) and systems of various type numbers (number of poles of G(s) at origin). A summary of the results of the above calculations may be seen in [1]. To facilitate the calculations, error coefficients are defined as

Position error coefficient, $K_p = \lim_{s\to 0} G(s)$

Velocity error coefficient, $K_v = \lim_{s\to 0} sG(s)$

Acceleration error coefficient, $K_a = \lim_{s\to 0} s^2 G(s)$

- The position control system has a second order transfer function in the standard form. (normalized form)
- The system should not have any steady state error for step input.
- The transient response of the system is affected by the value of K_A. A higher value of K_A should result in larger overshoot.

3.3 System Imperfections

All practical systems are imperfect to some extent. As a result of this, the actual system response differs from the ideal response of Fig. 5 (a), which is valid for a second order linear system. Some of the contributing factors relevant to the present set-up are :

(a) ***Saturation of magnetic field*** – the motor speed can not increase indefinitely with increasing control phase voltage. For large control phase voltages, therefore the motor shaft speed tends to saturate.

(b) ***Amplifier saturation*** - this implies limiting the maximum control effort for large errors leading to an upper limit of the shaft angular velocity.

(c) ***Dead zone*** - caused by a minimum voltage below which the motor would not start due to the friction of the bearings. As a result of this the steady state error may be larger than expected.

(d) ***Nonlinear motor characteristics*** - due to the inherent nonlinear torque-speed characteristics of an induction motor.

(e) ***System order*** - may be actually more than two, due to load characteristics, delays and filters used.

An accurate analysis taking into account the above mentioned imperfections would certainly prove to be exceedingly complex. The experiments which follow therefore consider the system as it is and study its performance. A qualitative comparison of the result of experiment with the theoretical predictions for a second order linear system should be of great interest.

4. EXPERIMENTAL WORK

The experiments suggested below enable the reader to study the performance of the closed loop system. Idea of dead zone and its effect on steady state error is also introduced. A special provision has been made in the set-up to store and display a response of the system - a need which occurs quite frequently. The operation of this waveform capture/display provision is described first.

4.1 Waveform Capture/Display

This card is designed to automatically store the response of the system in a RAM whenever a step input is given. The stored response is then displayed on the CRO. Steps for its operation are as given below :

(a) Power ON the system and/or press the RESET switch - unit goes into DISPLAY, drawing the axes and shows the RAM contents (zero at present).

(b) Press the MODE switch - the unit becomes ready to capture the step response.

(c) Applying step input now starts the storage. At the end of the capture cycle, the mode automatically shifts to DISPLAY and the response waveform is seen on the CRO.

(d) Storage of a new response or pressing the RESET switch erases the current waveform.

(e) The time scale of the display may be calibrated by feeding the X-output (sawtooth) of the unit to the Y-input of the CRO and determining its time period and amplitude.

4.2 Closed loop study (Also see the Note at end, on page 9)

(a) Position control through CONTINUOUS command

- Ensure that the step command switch is OFF
- Starting from one end, move the COMMAND potentiometer in small steps and observe the rotation of the response potentiometer.
- Record and plot θ_R, V_R, θ_0 and V_0 for a few values of K_A.
- Calculate $\Delta\theta_R$ and $\Delta\theta_0$ (taking initial readings as nominal values) and plot. Also calculate the errors ($\Delta\theta_R$-$\Delta\theta_0$), (ΔV_R-ΔV_0) at each step. Justify the presence of errors and their variation with K_A.

(b) Position control through STEP command

- Adjust the reference potentiometer to get $V_R = 0$ (around 10°)
- Set K_A to 2.
- Connect the CRO, calibrate the time scale, sec. 4.1(e), and switch to CAPTURE mode.
- Apply STEP input. Wait till storage is complete and the response is displayed. Trace the waveform from CRO.
- Compute M_p, ζ, t_p, t_r and the steady state error.
- Repeat for $K_A = 3,4,....$

- Tabulate the results as shown in the next section and discuss :
    - Variation of maximum overshoot, rise time and steady state error with forward gain.
    - Effect of dead zone and saturation on step response.

5. TYPICAL RESULTS

Typical results obtained on a similar unit are next given for guidance.

(a) Manual operation of the position control

$K_A = 5$

S. No.	θ_R deg	$\Delta\theta_R$ deg	θ_0 deg	$\Delta\theta_0$ deg	$\Delta\theta_R$-$\Delta\theta_0$ deg	V_R volt	V_0 volt	ΔV_R-ΔV_0 volt
1.	20°	--	20°	--	--	0.35 V	0.32 V	0.03 V
2.	50°	30°	53°	33°	3°	0.76 V	0.76 V	0.00 V
3.	80°	60°	85°	63°	3°	1.18 V	1.21 V	-0.03 V
4.	110°	90°	112°	92°	2°	1.57 V	1.59 V	-0.02 V
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(b) Calibration of X-output

In the DISPLAY mode with X-output connected to the Y-input of CRO, a sawtooth waveform is seen. On measurement,

Amplitude of sawtooth = 5.6 volts.

Time duration of the main linear part = 39 msec.

X-output scale factor is thus 6.96 msec/volt

The X-output waveform above consists of axis display part and waveform display part. The latter is identified by a much longer time duration which has been measured above.

(c) Step response of the position control

S. No.	K_A	M_p %	t_p msec	t_r msec	ζ	V_s volt	e_{ss} volt	ω_n Rad/sec
1.	2	--	--	--	--	2.50	0.20	--
2.	4	18	16.10	12.52	0.479	2.50	0.25	222
3.	6	22	13.92	9.74	0.434	2.50	0.20	250
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The values of time in the above table relates to the display time as calculated in (b) above. The physical system response is much slower. The two may be roughly correlated using a stop watch.

NOTE*: Due to various non-linearities in the system, viz., saturation of the amplifier, friction and backlash in gear, and most important the non-linear torque-speed characteristics of the motor, the usual behaviour expected in a linear position system may not be observed in all cases above. Specifically, with increasing values of gain K_A, the value of MP may not increase or the values of δ and e_{ss} may not decrease in all the readings tabulated above.*

Referring to Fig. 5,

$e_{ss} = V_s - V_0(\infty)$

$$M_P = \frac{V_{PEAK} - V_0(\infty)}{V_0(\infty)} \times 100\%$$

$$\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = 222\,\text{rad/sec} \qquad \zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}} = 0.479$$

t_p, t_r may be obtained from CRO

ζ may be calculated from M_p using the standard relation

$M_p = \exp\left(-\pi\zeta/\sqrt{(1-\zeta^2)}\right)$

ω_n is calculated from the expression of t_p $\{= \pi/\omega_n\sqrt{(1-\zeta^2)}\}$

The closed loop and open loop transfer functions of the system may now be written as,

$$\text{Closed loop : } \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{4.9\times10^4}{s^2 + 2.12\times10^2 s + 4.9\times10^4}$$

$$\text{open loop (excluding } K_A\text{) : } \frac{1}{K_A}\frac{\omega_n^2}{s(s+2\zeta\omega_n s)} = \frac{1}{4}\times\frac{4.9\times10^4}{s\left(s+2.12\times10^2 s\right)}$$

- The open loop transfer function (excluding K_A) comes out to be different for different readings - the system is not actually a second order function.
- The peaks of the response curves are flattened - the motor has dead zone.
- The peak overshoot does not increase significantly with K_A - motor current saturates.

NOTE*:* Under certain operating conditions, the motor may start continuous uncontrolled rotation, **this is not system oscillation.** *The basic cause is the small gap (approx. 5°) in the response potentiometer winding which is easily overshot by the motor, due to its inertia. In such a situation normal operation may be restored by decreasing the gain or by changing the position of the command potentiometer.*

6. REFERENCES

[1] Control System Engineering - I. J. Nagrath and M. Gopal, Wiley Eastern Ltd.

[2] Modern Control Engineering - K. Ogata, Prentice Hall of India Pvt. Ltd.

[3] Automatic Control Systems - B.C. Kuo, Prentice Hall of India Pvt. Ltd.

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STUDY OF AC SERVOMOTOR ACS-01

1. OBJECTIVE

To study the characteristics of a small a.c. servomotor and determine its transfer function.

2. SYSTEM DESCRIPTION

The unit in a self contained system for conducting the experiment except a measuring CRO which should be available in the laboratory. The different components of the unit are explained below.

(a) ***AC Servomotor*** – a 15W servomotor with identical reference and control phases operating at 12V/ 50Hz. Necessary phase shifting capacitor is pre-wired to the reference phase.

(b) ***Electrical load*** – in the form of a coupled dc generator and the required resistive load is provided.

(c) ***Time Constant*** – a special circuit to display the time-constant directly in milli-second.

(d) ***Metering*** – of all ac and dc voltage/currents is through built-in digital panel meters.

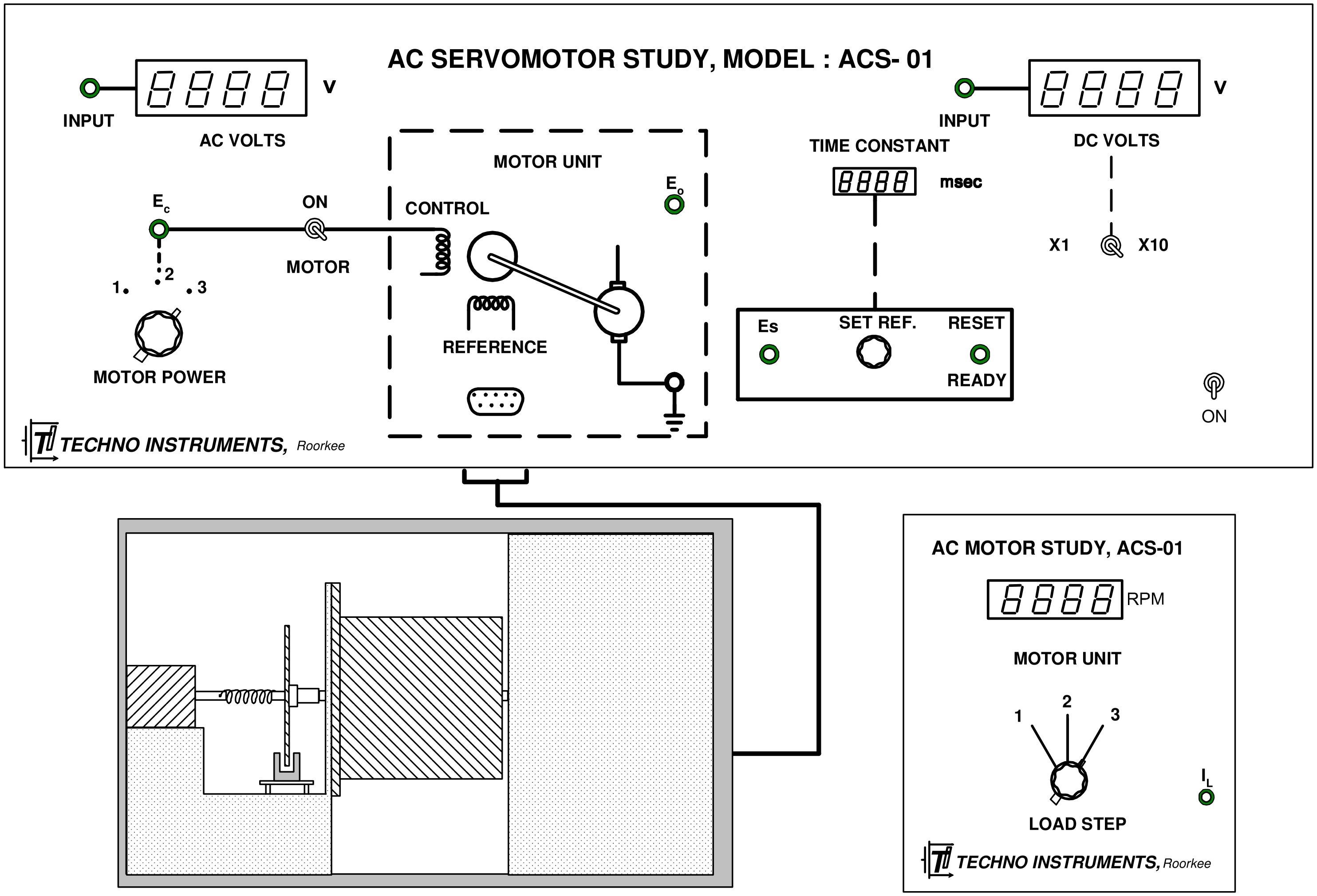
(e) ***Power supply*** – for conducting all experiments are available in the unit, which operates from a 220V/ 50Hz mains.

3. BACKGROUND SUMMARY

A.C. Servo Motors are basically two-phase, reversible, induction motors modified for servo operation. A schematic diagram of the motor is shown in Fig.1. The two windings, reference and control, may or may not have identical ratings. In the present unit both are rated at 12 volts r.m.s. at 50Hz. A phase shifting capacitor of appropriate value must be connected in series with one of the windings to produce a 90 degree phase shift.

These servo motors are used in applications requiring rapid and accurate response characteristics. A typical torque-speed characteristics of an induction motor is shown in fig.2 for two values of rotor resistance. A servomotor however must have negative slope in its torque-speed characteristics in order to ensure stable operation. To meet the above requirements, these ac servo motors have small diameter, light weight, low inertia and high resistance rotors. The motor's small diameter provides low inertia for fast starts, stops, and reversals. High resistance provides nearly linear torque-speed characteristics. A common structure is a drag-cup rotor. The a.c. servomotors have distinct advantages over d.c. servomotors. The commutator and brush assembly of a d.c. servomotor has limited maintenance free life. These are absent in the a.c.servomotor.

An induction motor designed for servo use is wound with two phases physically at right angles or in space quadrature . A fixed or reference winding is excited by a fixed voltage source, while the control winding is exited by an adjustable or variable control voltage, usually from a servo-amplifier. The servo motor windings are often designed with the same voltage/turns ratio, so that power inputs at maximum fixed phases excitation, and at maximum control phase signal, are in balance. In the present unit the input to the control winding is adjustable (3-steps) and the motor can be switched 'ON' through a switch.



Panel drawing AC Servomotor Study, ACS - 01

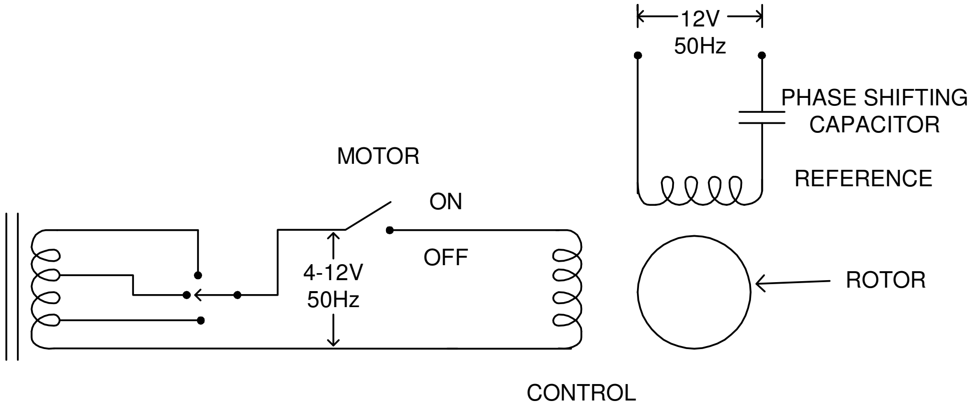


Fig 1 : 2-Phase A.C. Servomotor

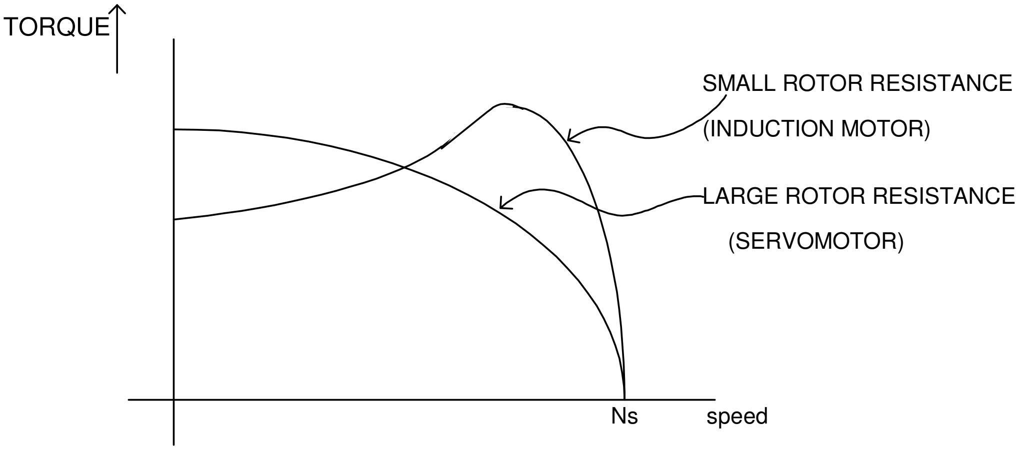


Fig 2 : Torque-speed characteristics of induction motors

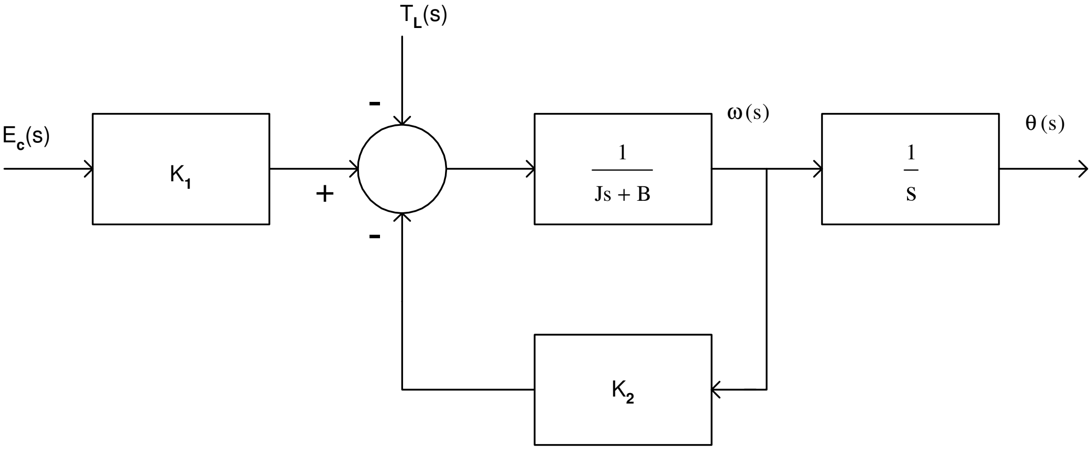


Fig 3 : Block diagram of an a.c.motorsystem

The block diagram of an a.c. servomotor system is presented in Fig.3 This is a highly simplified and linearized version of the actual behaviour of the motor and is valid at low speed of operation only. Detailed description of the working and derivation of the block diagram may be seen in any of the text books listed at the end of this document. Here E_c is the voltage applied at the control phase which results in a proportional torque which however is reduced by a factor related to the motor torque efficiency to generate the actual motor torque as

$T_m (s) = K_1 E_c(s) - K_2 \omega(s)$, where ω is the shaft speed .

This torque, further reduced by the mechanical load torque, T_L, drives the inertia, J and the friction, B of the motor to result in the speed, ω and subsequently the angular position, θ of the motor shaft.

From the block diagram of Fig.3 the transfer function of the motor may be written as

$$\frac{\theta(s)}{E_c(s)} = \frac{Km}{s(\tau_m s+1)} \quad \text{for } T_L(s) \equiv 0 \qquad (1)$$

where, $K_m = \dfrac{K_1}{B+K_2}$, and $\tau_m = \dfrac{J}{B+K_2}$,

are the motor gain constant and the motor time constant respectively. As students of control system, our interest is to evaluate the transfer function and the parameters of the ac servomotor.

Again for $E_c(s) \equiv 0$,

$$\frac{\theta(s)}{T_L(s)} = -\frac{\frac{1}{B+K_2}}{s(\tau_m s+1)} = -\frac{K_n}{\tau_m s+1}, \text{ where } K_n = \frac{1}{B+K_2}$$

Combining the above two transfer functions (under assumption of linearity),

$$s\theta(s) = \omega(s) = \frac{K_m}{(\tau_m s+1)}E_c(s) - \frac{K_n}{(\tau_m s+1)}T_L(s)$$

The computation of K_m and K_n can be done by using the final value theorem, i.e.,

Steady state speed, $\omega_{ss} = \lim_{s\to 0} s\omega(s) = K_m E_c - K_n T_L$ (2)

where

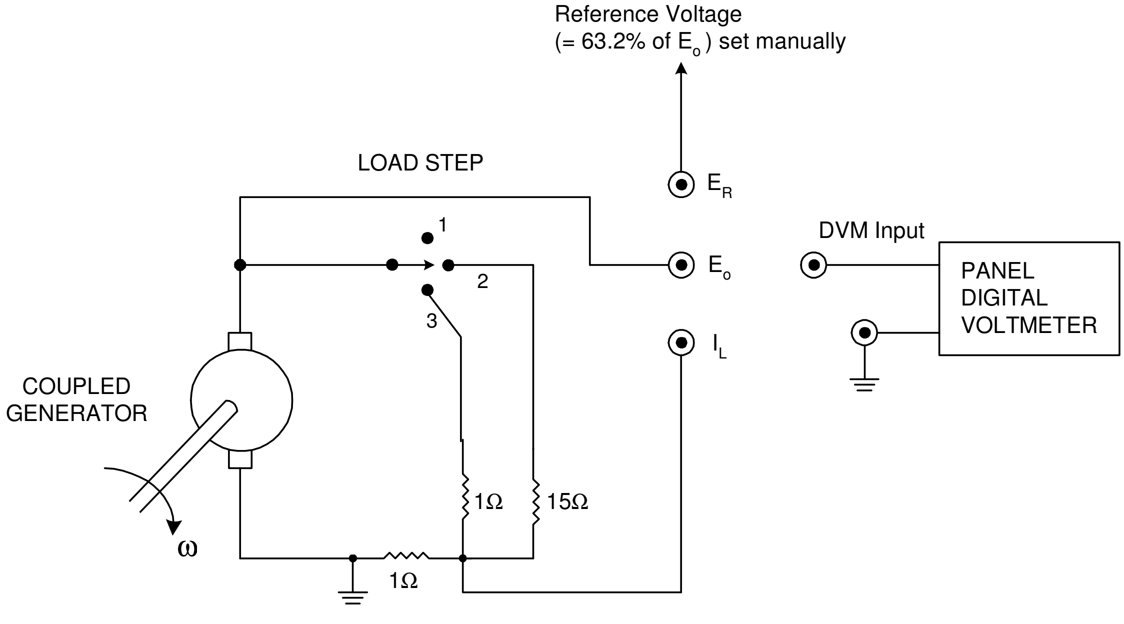
E_c = Constant voltage applied to the control winding

T_L = Constant Load torque

E_c is measured by the a.c. voltmeter on the panel and T_L is calculated from the loading of the coupled d.c. generator as ,

$$T_L = \frac{\text{Electrical Power drawn from the generator in watts}}{\text{Angular velocity of the shaft in radians / sec}}$$

The calculation assumes that the generator mechanical parameters are negligible compared to that of the servomotor.



DVM input to be externally connected to terminal
E_o To measure generator output voltage
E_R To measure 63.2% of E_o for Time-constant measurement
I_L To measure load current as the drop across 1Ω resistance

Fig : 4 Loading circuit arrangement

All the above results are strictly valid if the system is perfectly linear. This is true to a great extent, especially at low speeds, and will form the basis of conducting the present experiment. Another option, though cumbersome, is to determine experimentally the torque-speed and torque-control phase voltage characteristics and then to linearize these graphically to evaluate the motor parameters, K_1 and K_2, and then to calculate K_m, τ_m, J and B.

4. EXPERIMENTAL WORK

The a.c. motor study is divided into groups (a) steady state – to determine K_m and K_n, and (b) transient – to determine τ_m. From these the transfer function and other constants are calculated.

4.1 Steady State Operation

4.1.1 Determination of Generator Constant

The generator constant, K_G, in volts/rpm, may be computed from the no load generator voltage data at various speeds. This would enable one to calculate the generated voltage under loaded condition, which is needed for torque computation in the next section. The readings for the present experiment may be tabulated as below:

TABLE – 1

INPUT STEP	MOTOR SPEED, N_r, RPM	LOAD STEP-1 (No Load) Voltage, Volts	GENERATOR CONSTANT, K_G, Volts/rpm
1.	1255 rpm	2.63 volt	0.00209 Volts/rpm
2.	1869 rpm	3.84 volt	0.00205 Volts/rpm
3.	1928 rpm	3.99 volt	0.00207 Volts/rpm

Average Generator Constant, $K_G = 2.07 \times 10^{-3}$ Volts/rpm

4.1.2 Determination of Motor Parameters

The motor is operated at various combination of control phase voltage, E_c, and external loading, T_L, and the data is recorded as in Table-2. E_c is measured with the help of the a.c. voltmeter on the panel in three steps while no load generator voltage, E_0 and load currents, I_L are measured by a switchable d.c. panel meter provided. The loading circuit is shown in Fig. 4

Operating at no load (load step-1), ie., $T_L = 0$, using equation (2) the motor gain constant may be calculated as

$$K_M = \frac{\omega_{ss}}{E_C} = \frac{N\pi}{30 E_C}$$

An average value for K_m may be obtained from the three input voltage steps provided.

K_{m1}=54.31 rad/ volt-sec

K_{m2}=27.53 rad/ volt-sec

K_{m3}=18.77 rad/ volt-sec

TABLE – 2

INPUT		LOAD STEP –1 (no load)				LOAD STEP -2			LOAD STEP-3		
STEP	E_c, rms V	E_0, Volts	I_L, amp	N, rpm	T_L= $(30.E_0.I_L/N.\pi)$	I_L	N	$T_L = \frac{(30K_G I_L)}{\pi}$	I_L	N	T_L
1	3.80	2.66	0	1447	0	0.51	667	0.0010	0.067	470	0.0013
2	7.50	3.84	0	1911	0	0.94	1254	0.0018	0.135	894	0.0026
3	11.00	4.00	0	1972	0	0.11	1488	0.0021	0.168	1139	0.0033

$$K_m = \frac{K_{m1} + K_{m2} + K_{m3}}{3} = 33.53 \ \text{rad/volt} - \text{sec}$$

An average value for K_m may be obtained from the three input voltage steps provided although the three values of K_m obtained are very different from each other due to the motor non-linearity.

Similarly, operating at a constant E_c and two different load steps, one gets

$$\omega_{ss1} = K_m E_c - K_n T_{L1}$$

$$\omega_{ss2} = K_m E_c - K_n T_{L2}$$

From these, K_n may be obtained as,

$$K_n = \frac{\omega_{ss1} - \omega_{ss2}}{T_{L2} - T_{L1}}$$

or, the effective friction as

$$B + K_2 = \frac{T_{L2} - T_{L1}}{\omega_{ss1} - \omega_{ss2}}$$

The inertia, J, may further be calculated from

$J = \tau_m . (B+K_2)$, after τ_m is computed as outlined in sec, 4.2

4.2 Transient Operation

The time constant, τ_m is the time taken by the motor to reach 63.2% of the steady state speed when a step voltage is switched on the control winding while the reference winding is already excited at the rated voltage. In the present unit this is achieved through a special circuit which displays the time constant in milli-seconds. The steps for operating this circuit are given below:-

Step. 1. Switch the motor 'ON' at input step.3 (rated voltage). A constant speed will be indicated almost immediately.

Step .2. Read V_0 at Load step-1. Set 'REFERENCE' potentiometer in the TIME CONSTANT SECTION to 63.2% of the E_0 value read above. Use the d.c. voltmeter in the SET REF position.

Step. 3. Switch the motor 'OFF', wait for 30 seconds and then switch it to 'ON' position. The time constant will be displayed in msec.

The time constant obtained above may have error due to non-linear friction present. It is therefore desirable to conduct the experiment a number of times and average the result.

5. RESULTS AND DISCUSSIONS

The results given below taken from a sample unit for input-3 and may differ from the unit supplied to you. This is because of the variation in the characteristics of the motor, generator and to some extent on the experimental errors.

(a) Reference winding input, E_c = 12.0 V, rms
Motor speed, N = 1928 rpm
Generator coefficient, K_G = 2.07×10^{-3} volts/ rpm

(b) Reference winding input, E_c = 12.0 V, rms

Motor speed, N = 1972 rpm

Motor gain constant,

$$K_m = \frac{N\pi}{30E_c} \text{ rad/v-sec} = 18.77 \text{ rad/v-sec}$$

(c) Reference winding input, E_c = 12.0 V, rms

Motor time constant, τ_m = 110 msec

(d) Transfer Function of the motor

$$G(s) = \frac{K_m}{s(s\tau_m + 1)} = \frac{18.77}{s(0.11s + 1)}$$

The typical results shown above are all based on averaged set of data. These are primarily aimed at showing the method. The students are expected to complete the experiment in detail.

Further, the above experiment computes the motor transfer function only, which is probably the most significant result. In addition to this, the students may compute inertia J and effective friction ($B + K_2$) as well, as outlined in section 4.12

The most important assumption in this experiment is that of considering the system to be linear. Strictly speaking the a.c.servomotor and the d.c. generator are both non-linear. Further, there are non-linear friction components which have been totally neglected. All these contribute to errors in the result although the transfer function derived represents the actual behaviour of the motor pretty well, at least at low speeds

6. REFERENCE

[1] M. Gopal, "Control Systems – Principle and Design", Second Edition, Tata McGraw Hill Publishing Co. Ltd., pp.168-174.

Laboratory Manual

for

Control System Lab

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COMPENSATION DESIGN CD-02

1. OBJECTIVE

To design, implement and study the effects of different cascade compensation networks for a given system.

2. EQUIPMENT DESCRIPTION

The unit has been designed with the objective of exposing the students to the problem of control system compensation. A simulated system of 'unknown dynamics' is available which may be studied both in the time and frequency domains. In addition, the forward gain is variable, thus the system dynamics is adjustable in a wide range as well. The closed-loop system presents an 'unsatisfactory' performance. A set of performance specifications is to be prescribed by the teacher, and the student would design a suitable compensator. Necessary theoretical background and design steps are explained in section 3, covering the s-plane and ω -plane designs of both lag and lead networks. Compensation network so designed may be easily implemented in the unit, and its effect on the performance may be evaluated. All necessary facilities are built-in in the system. Only a measuring CRO and a few passive components are needed for conducting the experiment. Referring to the block diagram of Fig. 1, the various sections of the unit are described below in some detail.

2.1 Signal Sources

There are three built-in sources in the unit having the following specifications :

(a) Sine wave:	Smoothly adjustable frequency from 25 Hz to 800 Hz in a single range. Amplitude: 0-1 Volt p-p (variable)
(b) Square wave:	Smoothly adjustable frequency from 25 Hz to 800 Hz in a single range. Amplitude 0-1 Volt p-p (variable)
(c) Trigger:	At the frequencies set above.

The frequency is displayed on a 4-digit frequency meter on the panel

All the above sources are derived from the same basic circuit and are therefore synchronized. They are calibrated in frequency but uncalibrated in amplitude, are zero balanced, and have a common ground.

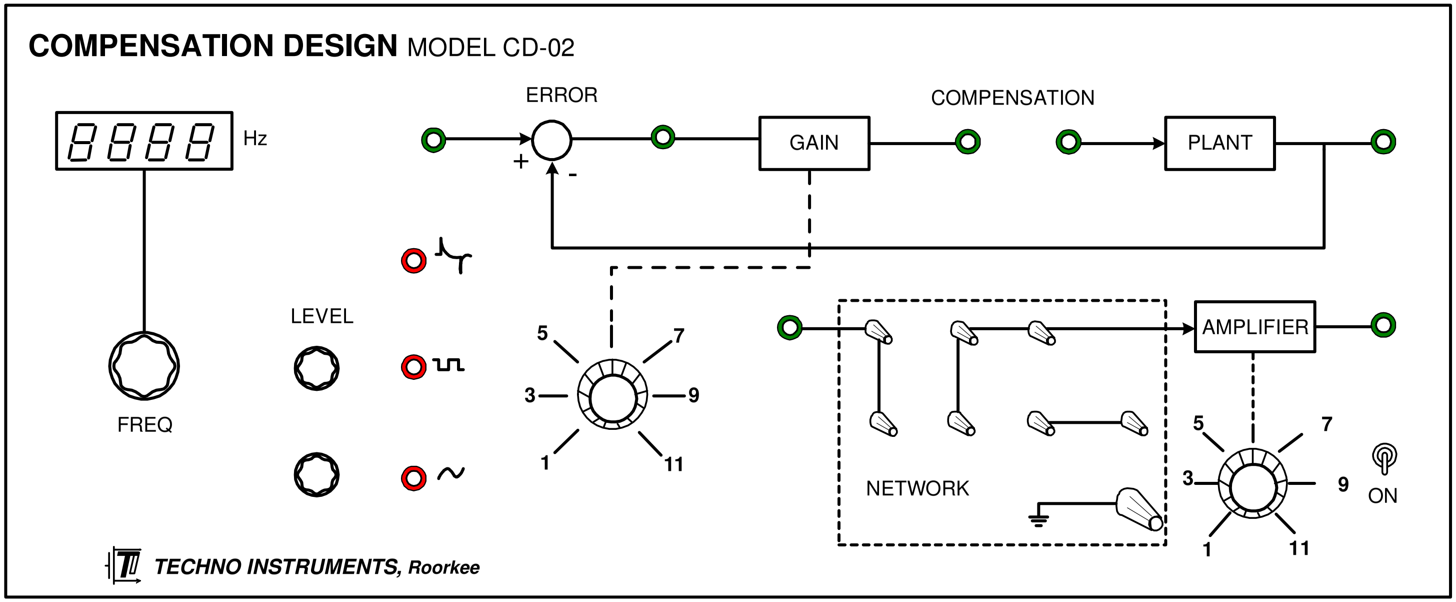
2.2 Uncompensated System

A simulated system of 'unknown dynamics' forms the uncompensated system. The circuits are pre-wired except for terminals in the open loop where a compensation network may be inserted. The two sections of this system are :

(a) ***Plant***: It is an active network simulation of a second order dynamic system. Its transfer function is given by

$$\frac{K_1}{(sT+1)^2}$$

The values of K_1 and T are not explicitly given but are determined through experimentation.



Panel Drawing of Compensation Design, Model CD-02

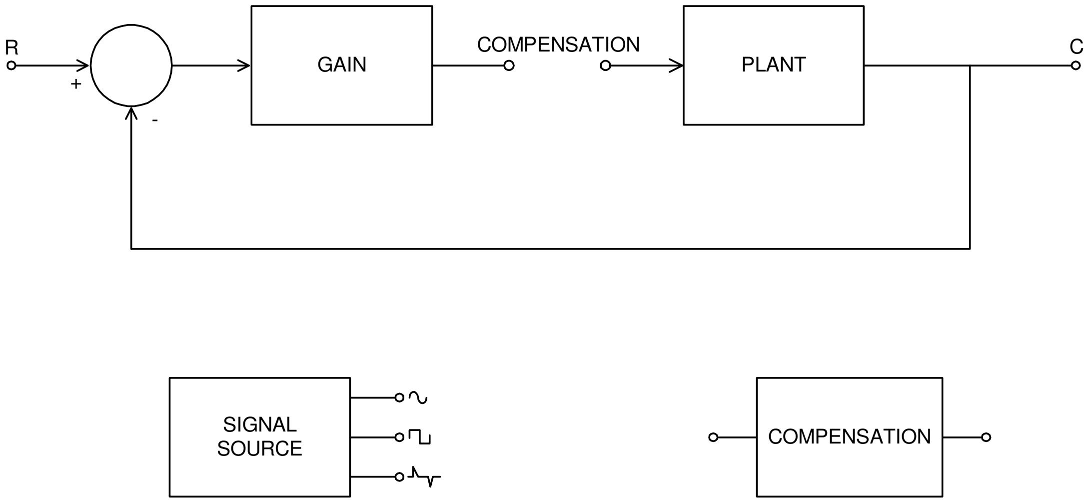


Fig.1. System Block Diagram

(b) ***Error detector-cum-gain***: This block has two inputs (e_1,e_2) and an output (e_o) related by the expression, $e_o = K (e_1+e_2)$, where K is a variable gain. The value of K may be varied from 1 to 11 and may be read on the dial.

2.3 Compensation Circuit

This circuit consists of a pre-wired variable gain amplifier where-in the gain may be varied from 1 to 11 and read on a dial. The circuit has provision for connecting a few passive components in accordance with the design of compensator.

2.4 Power Supply

The set up has an internal ±12V IC regulated supply which is permanently connected to all the circuits. A separate internal 5 Volt supply powers the frequency meter. The power supply and all circuits are short circuit protected and will not get damaged even if wrongly connected. No external DC supply should, however, be connected to the unit.

The above set up can be used to study the improvements caused by a variety of compensation networks. Constraints on time available would however limit such study to only one network in a normal laboratory period of 3 hours.

3. BACKGROUND SUMMARY

Practical control systems use a range of mechanical, electrical, hydraulic, thermal and other type of components for their operation. Examples include motors, gears, amplifiers, control valves, heat exchangers etc. The design of these components is usually based upon requirements other than those which might be prescribed by the control engineer - for example a specified transfer function. As a result, the control engineer is constrained to make the best possible choice out of the components offered by the manufacturer. The system so constructed may not be entirely satisfactory. Compensation network is designed at this stage to modify the system characteristics and to force it to meet the specifications. Although compensation elements are used at the output (load compensation) and in the feedback path (feedback compensation), the most common form of compensation is the cascade compensation where the compensation acts on the error signal. The principal advantage of this configuration is that the signal level of the error is very low and the error is more commonly electrical in nature. Thus the compensation network needs to be a low power electrical network which is very easy to implement. Basic theory of compensation is discussed in the following pages. However, a thorough understanding of control system analysis is a prerequisite for this experiment.

3.1 Performance Specifications

Before taking up the design of a compensation network, it is necessary to evaluate the performance of existing uncompensated system. This is done in terms of a number of performance criterion which provide quantitative idea of the system performance. The design of compensation network may be carried out either in the s-plane, through root locus diagram, or in the frequency domain, using the Bode plot, Nyquist diagram and Nichol's chart. The usual performance criterion applicable to the two approaches are given below.

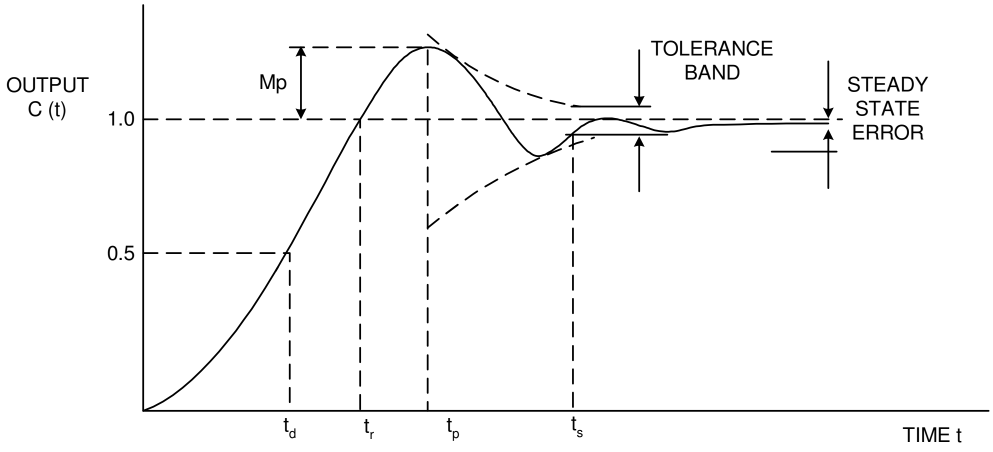
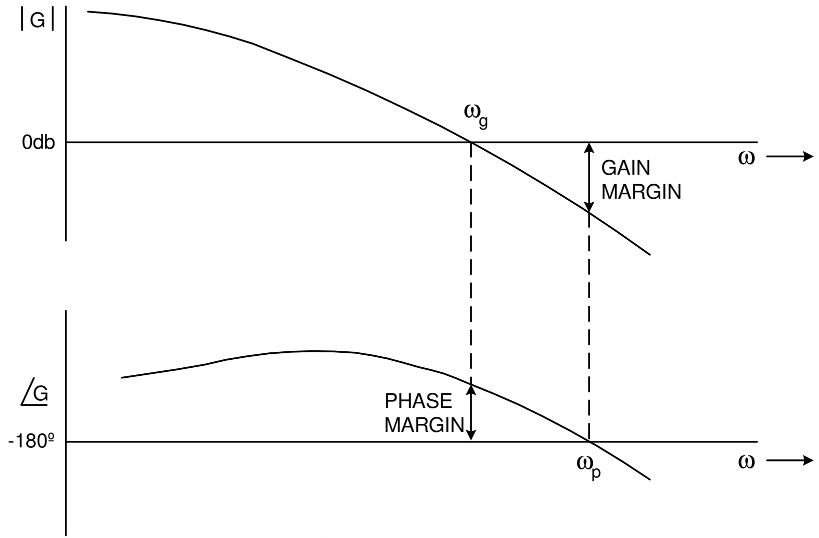
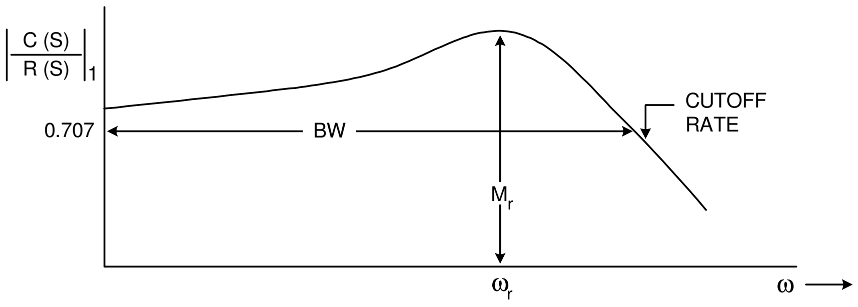


Fig. 2 Time Domain Performance Criterion



(a) Open Loop Bode Plot



(b) Closed Loop Frequency Response

Fig. 3. Frequency Domain Performance Sriterion

(a) Time-domain performance criterion: These are indicative of the performance of the close-loop system in terms of its time response, most commonly the unit step response. Since a control system is almost always required to function in real time, time-domain performance criterion is a direct way of evaluating the system. Due to one-to-one correlation between s-plane pole location and the resulting step response, the time-domain performance criterion finds application in the root locus method of analysis and design. Referring to the unit step response shown in Fig. 2, the various time domain performance criterion are :

(i) *Delay time* t_d, defined as the time needed for the response to reach 50% of the final value

(ii) *Rise time* t_r, the time needed for the response to reach 100% of the final value for the first time

(iii) *Peak time* t_p, the time taken for the response to reach the first peak of the overshoot

(iv) *Maximum Overshoot* M_P, given by

$$M_P = \frac{c(t_P) - c(\infty)}{c(\infty)} \times 100\%$$

(Its value indicates the relative stability of the system)

(v) *Settling time* t_S, the time required by the system step response to reach and stay within a specified tolerance band which is usually taken as ± 2% or ± 5%

(vi) *Steady state error* e_{SS} defined as

$$e_{SS} = \lim_{t \to \infty} [r(t) - c(t)]$$

The above performance criterion are very general and are valid for systems of any order, however, their mathematical expression in terms of system parameters are available only for a second order system. Root locus design therefore essentially relies on the assumption that the system in question is of second order or approximately second order.

It may further be pointed out that all the above specifications may not be satisfied in a given problem unless these are consistent. Usually one steady state specification and one transient specification is required to be met by the system.

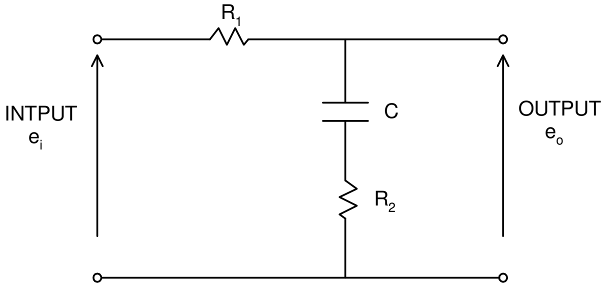
(b) Frequency-domain performance criterion: These are specifications indicated on the open loop frequency response curves of the system i.e. Bode plot, Nyquist diagram or Gain magnitude - phase shift plot or the closed loop frequency response of the system. Unlike the time-domain specification, a number of the frequency-domain criterion are defined on the closed loop characteristics. Frequency-domain approach to design is easily applicable to systems with unknown dynamics by experimentally determining the frequency response. However, the correlation between time and frequency response is not direct.

Referring to Fig. 3(a) (open loop Bode plot) and Fig. 3(b) (closed-loop frequency response), the various performance criterion in the frequency-domain are given below:

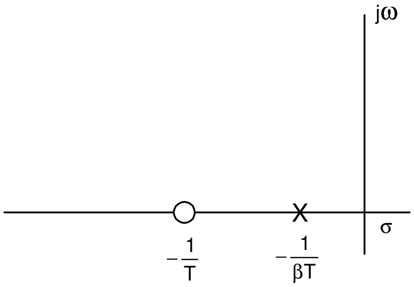
(i) *Gain Margin*, is the amount by which the open loop gain may be increased at the phase cross over frequency, ω_P, to bring it to 0 dB.

(ii) *Phase Margin*, is the amount by which the phase of the open loop transfer function at the gain cross over frequency, ω_g, may be increased in the negative direction to bring it to -180°.

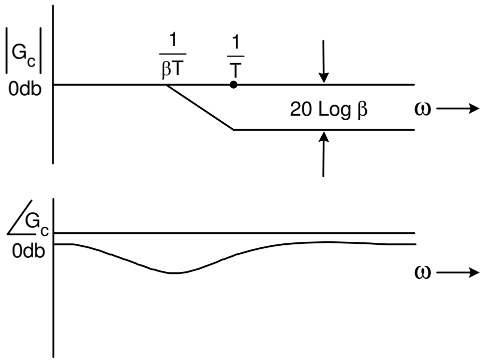
(iii) M_r, Peak value of closed loop frequency response.



(a) Network Structure



(b) Pole-Zero Configuration



(c) Bode Diagram

Fig. 4 Lag Network Characteristics

(iv) ω_r, Frequency at which the peak occurs.

(v) *Bandwidth* of the closed loop frequency response.

(vi) *Cut-off rate* of the closed loop frequency response at the high frequency end.

All the above specifications may not be satisfied in a given problem unless these are consistent. Usually the steady state error along with phase margin specifications is required to be satisfied.

3.2 Design Philosophy

In a control system, the forward path gain K is frequently adjustable. In general, therefore, the gain may be chosen such that the system either satisfies the steady state specification or the transient specification, but not both. The design of the compensation network must then ensure that the other specification is now met without disturbing the first. The most common form of compensation network is an R-C passive network having a pole and a zero. This gives rise to 'lag' and 'lead' network depending upon the relative locations of the pole and the zero. The characteristics of these networks are described below in some detail.

Lag network: The lag network is shown in Fig. 4(a). Its transfer function may be expressed as

$$G_c(s) = \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1}$$

Substituting $R_2C = T$, and $(R_1+R_2)/R_2 = \beta$ (>1), $G_c(s)$ may be written in two alternative forms :

$$G_c(s) = \frac{Ts + 1}{\beta Ts + 1} \qquad \ldots\text{Form I}$$

$$= \frac{1}{\beta} \cdot \frac{(s + 1/T)}{(s + 1/\beta T)} \qquad \ldots\text{Form II}$$

Form I is directly suitable for frequency-domain design while Form II with the $1/\beta$ factor cancelled by an amplifier, is suitable for root locus design. Pole zero configuration and the Bode diagram of the lag network are shown in Fig. 4(b) and (c) respectively. It may be noted that the network exhibits a low pass character and introduces a negative phase angle.

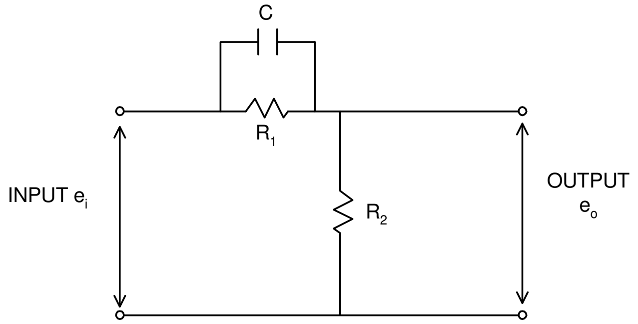
Lead network: The lead network is shown in Fig. 5(a). Its transfer function may be expressed as

$$G_c(s) = \frac{R_2}{R_1 + R_2} \cdot \frac{R_1Cs + 1}{\frac{R_1R_2\,Cs}{R_1 + R_2} + 1}$$

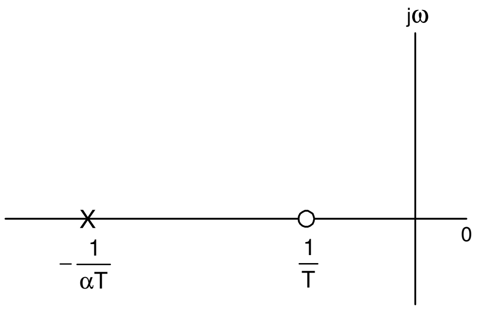
Substituting $R_1C = T$, and $R_2/(R_1+R_2) = \alpha$ (<1), $G_c(s)$ may be written in two alternative forms:

$$G_c(s) = \frac{\alpha\,(sT + 1)}{(s\alpha T + 1)} \qquad \ldots\text{Form I}$$

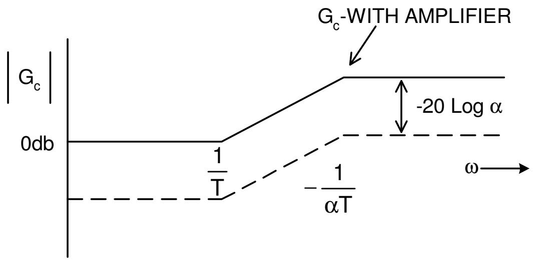
$$= \frac{s + 1/T}{s + 1/\alpha T} \qquad \ldots\text{Form II}$$



(a) Network Structure



(b) Pole-Zero Configuration



(c) Bode Diagram

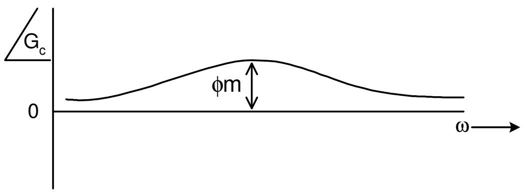


Fig. 5 Lead Network Characteristics

Form I with the factor α cancelled by an amplifier is suitable for frequency domain design, while Form II is directly suitable for root locus design. Pole zero configuration and the Bode diagram of the lead network are shown in Fig. 5(b) and (c) respectively. It may be noted that the lead network exhibits a high pass character and has a positive phase angle. It may be further shown that the maximum lead angle ϕ_m produced by this network is given by

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}, \text{ and is at the frequency, } \omega_m = \frac{1}{\sqrt{\alpha T}}$$

Using these networks, the design may be carried out either in the frequency domain or in the s-plane as described below :

(a) ***Design in the frequency domain***: The design here begins with the Bode diagram of the open loop system where the forward path gain has been adjusted so as to satisfy steady-state requirements in terms of e_{ss} or error coefficients. Transient specification, usually given in terms of the phase margin, is now checked. An improvement in the phase margin can be attempted by one of the following methods :

(i) If required phase margin is likely to be obtained at a lower frequency, the gain cross-over frequency may be brought down by using the high frequency attenuation of the lag network.

Note that the lag network does not disturb the low frequency gain set earlier. Care needs to be exercised to ensure that the negative phase angle of the lag network does not affect the phase margin substantially.

(ii) A positive phase angle may be added by inserting a phase lead network so that the phase margin improves. This requires a phase lead network with amplifier **so that the low frequency gain of the system remains unaltered.** Also, due to the high pass character of this network, the gain cross-over frequency has a tendency to shift to higher frequencies causing a lowering of the phase margin. This fact too needs to be taken care of in the design.

(b) ***Design in the s-plane***: The s-plane design starts with a root locus sketch of the system. Thus the transfer function of the system must either be available or it should be computed from the experimental frequency response data. For this design, specifications may include steady-state error e_{ss} or error coefficients and a transient response specification in terms of peak overshoot/damping ratio/settling time. Based on the transient response specifications, the desired closed loop pole location in the s-plane is marked. Following two possibilities now exist :

(i) If the root locus passes through the desired location determined above then the value of K is selected so as to place the closed loop poles at the proper location. Next the factor by which K_v, the velocity error coefficient, (or K_p, K_a etc. depending on the type number) needs to be multiplied for steady state specifications to be met, is calculated. This multiplication of K_v (by β) is effected by the lag compensator transfer function of Form II when the $1/\beta$ factor is cancelled by an amplifier of gain β. The value of T is chosen so that a very small shifting of the root locus diagram is caused by the negative phase angle of the lag compensator.

(ii) If the root locus diagram does not pass through the desired location then a positive phase angle must be added to force the root locus diagram to pass through it. This is achieved by a lead network with appropriately chosen poles and zero. Finally the value of steady-state error is evaluated and minor adjustment in the compensator pole/zero is made as necessary.

4. EXPERIMENTAL WORK

All the four compensation design problems outlined above may be attempted on the present setup unless the performance specification chosen are outside the physical limitations of the system. A substantial amount of experimentation and graphical design is involved in each, which is rather time consuming. It is therefore recommended that only one network may be designed and tested in a usual laboratory class. In the following pages experimental procedure for frequency domain design is presented. A reader having adequate knowledge of s-plane methods may however undertake root locus design as well.

4.1 Bode Plot of the Plant

As a first step the magnitude-frequency and phase-frequency plots are to be sketched from experimental data.

- Disconnect the COMPENSATION terminals and apply an input, say 1 V p-p, to the plant from the built in sine wave source. Vary the frequency in steps and calculate plant gain in dB and phase angle in degrees at each frequency. Sketch the Bode diagram on a semilog graph paper.
- From the low frequency end of the magnitude plot, obtain the error coefficient and the steady state error.
- Calculate the forward path gain K necessary to meet the steady state error specifications.
- Set the above value of K, short the COMPENSATION terminals and observe the step response of the closed loop system. Compute the time-domain performance specifications, namely, M_P, t_P, e_{SS} and ζ.
- Shift the magnitude by 20 $\log_{10}$ (K), and obtain the value of phase margin. Compare with the given specifications of phase margin.

4.2 Lag Network Design

- From the Bode plot, find a frequency where $PM_{actual} = PM_{specified}$ + a safety margin (5°-10°). This is new gain cross-over frequency $\omega_{g,new}$.
- Measure gain at $\omega_{g,new}$. This must equal the high frequency attenuation of the lag network, i.e. 20 log β. Compute β.
- Choose Z_c=1/T, at approx. 0.1 $\omega_{g,new}$ and P_c=1/βT accordingly.
- Write the transfer function $G_c(s)$ and calculate R_1, R_2 and C.
- Implement $G_c(s)$ with the help of a few passive components and the amplifier provided for this purpose. The gain of the amplifier must be set at unity.
- Insert the compensator and determine experimentally the phase margin of the plant.
- Observe the step response of the compensated system. Obtain the values of M_P, t_P, e_{SS} and ζ.

4.3 Lead Network Design

- From the Bode diagram obtained in section 4.1, calculate the required phase lead as

  Phase lead needed (ϕ_m) = $PM_{specified}$ - $PM_{available}$ + safety margin (5° to 10°)
- Calculate α for the lead network from

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

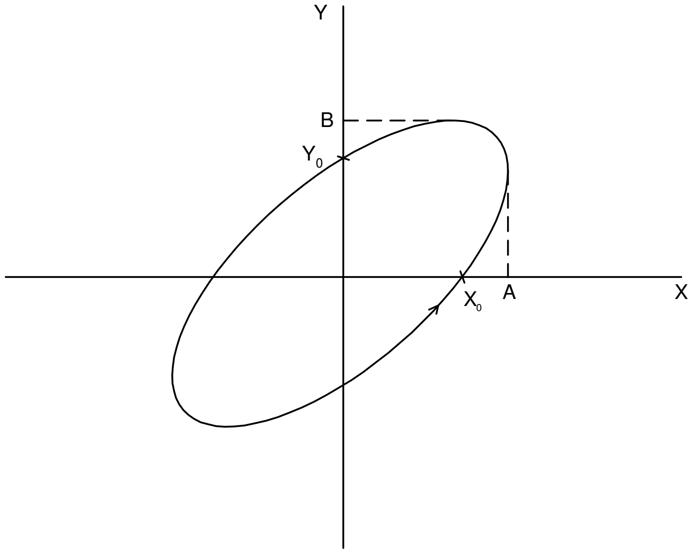


Fig. 6 Phase and Gain Measurement on CRO

- Calculate new gain cross-over frequency $\omega_{g,new}$ such that

  $$|G|_{\omega_{g,new}} = 10 \log \alpha$$

This step ensures that maximum phase lead shall be added at the new gain cross-over frequency.

- The corner frequencies are now calculated from $1/T=\sqrt{\alpha}\,\omega_m$ and $1/\alpha T=\omega_m/\sqrt{\alpha}$
- Implement $G_c(s)$ with the help of a few passive components and the amplifier provided for this purpose. The gain of the amplifier is to be set equal to $1/\alpha$.
- Insert the compensator and determine experimentally the phase margin of the plant with compensator.
- Observe the step response of the compensated system. Obtain the values of M_P, t_P, e_{ss} and ζ.

In addition to the above experiments, the measurement of frequency response of closed loop system, both before and after compensation, would provide further insight.

5. TYPICAL RESULTS

Following are the results obtained on a typical unit.

(a) Frequency response measurements :

Input = 1 Volt p-p, sine wave; CRO in x-y mode

All measurements are in volts p-p

f_{Hz}	A	B	x_o	y_o	Gain dB	Phase in degrees
16	0.5	2.2	0.1	0.4	12.86	- 10.4
31	0.5	2.0	0.2	0.8	12.04	- 23.58
40	0.5	1.65	0.4	1.35	10.37	- 54.9
80	0.5	0.5	0.48	0.90	5.57	- 71.3
100	0.5	0.72	0.46	0.68	3.17	- 109.2
200	0.5	0.25	0.3	0.16	- 6.02	- 140.2
300	0.5	0.12	0.22	0.05	- 12.4	- 155.4
400	0.5	0.07	0.16	0.024	- 17.07	- 160.0
800	0.5	0.017	0.08	0.035	- 29.37	- 168.1

These measurements are carried out by the ellipse method (or by a double trace CRO). If the input and output of a system, given as input $x = A\cos(\omega t)$, and output $y = B\cos(\omega t-\theta)$, are fed to the x and y plates of the CRO respectively, the resulting trace is an ellipse (Fig. 6) given by

$$y^2 + (B^2/A^2)x^2 - 2(B/A)xy \cos\theta = B^2 \sin^2\theta$$

Measurements of intercepts on x and y axes and peak values in these directions yield

$$\text{Gain} = B/A = y_o/x_o;\ \text{or } 20 \log (B/A) \text{ dB, and}$$

$$\text{Phase } \theta = -\sin^{-1}(x_o/A) = -\sin^{-1}(y_o/B)$$

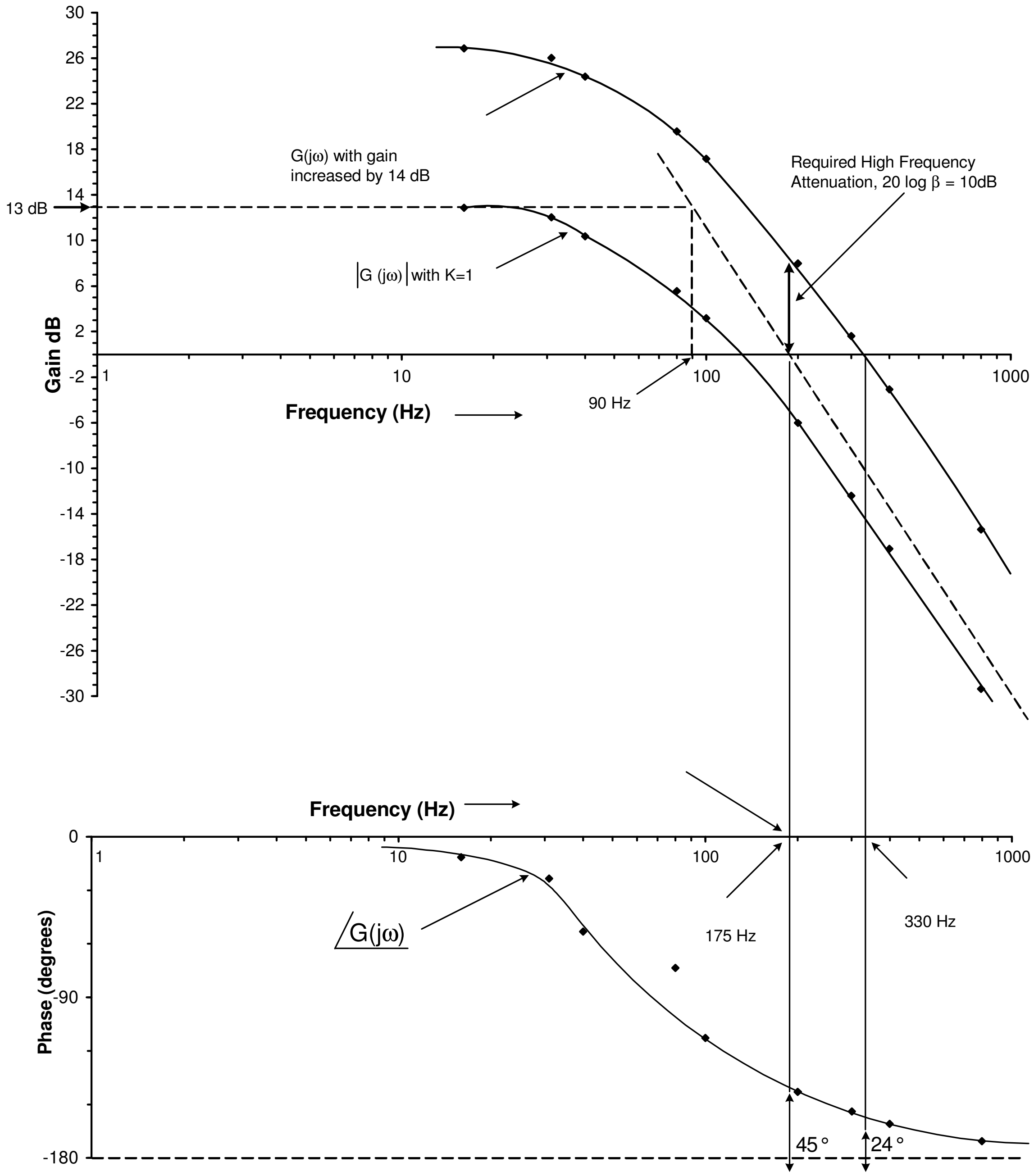


Fig. 7 Lag Compensator Design Example

It is easy to see that for 90°<θ<180°, the major axis of the ellipse of Fig. 6 has a negative slope and the phase angle is computed as $\theta = -180 + \sin^{-1}(x_o/A)$.

(b) Bode plot: For the readings obtained above, the Bode plot is shown in Fig. 7. From it, the approximate transfer function of the open loop system is obtained by drawing the low and high frequency asymptotes and observing the values of low-frequency gain and corner frequency. In the present set-up,

Corner frequency, f = 90 Hz, which gives $T = \dfrac{1}{2\pi.90} = 0.00176$

and gain at low frequency = 13db, which gives $K_1 = \text{antilog}\left(\dfrac{13}{20}\right) = 4.466$

Thus, Plant Transfer Function is obtained as,

$$\frac{4.466}{(1+0.001768s)^2}$$

(c) Design: ***(Lag network)***

Let the design requirement be

$e_{SS} = 0.05 \ (5\%)$

Phase margin = 40°

Required value of error coefficient K_P to meet e_{SS} specification is 19. Thus gain K needs to be increased by 19/4.466 = 5(say), i.e. by 20 log(5) = 14 dB (approx.). With this value of open loop gain, step response of closed loop system {Fig. I(a) of Appemdix-I} gives

$M_P = 50\ \%$
$t_p = 1.6$ m sec.
$e_{SS} = 4\ \%$

The gain setting of 5 must not be changed throughout this experiment.

The magnitude plot is redrawn by shifting up by +14 dB. From this plot

Gain cross over frequency $\omega_g = 2\pi\ (330)$
Phase Margin = 24°

Now, phase margin needed = 40° + 5° (safety margin) = 45°

This is available naturally at $\omega = 2\pi(175)$ which may be chosen as $\omega_{g,new}$.

High frequency attenuation needed = 10dB = 20log β, so that the value of β is 3.16. Compensation network is thus chosen as

$^*\omega_{g,new} = \omega_{g/10} = 2\pi(175)/10 = 1/T$

$Z_C = 1/T = 2\pi(17.5)$, $P_C = 1/\beta T = 2\pi(17.5)/3.16$

$T = R_2C = 0.009094$; $\beta = (R_1+R_2)/R_2 = 3.16$

Component values for implementation

$R_1 = 19.64\ k\Omega \approx 20\ k\Omega$
$R_2 = 9.09\ k\Omega \approx 9.1\ k\Omega$
$C = 1\mu F$

* **Note:** The corner frequencies of the network are placed sufficiently lower than the desired cross-over frequency, so that the phase lag contribution of the network at this cross over frequency is made small. Usually the upper corner frequency of the network is placed 1/8 or 1/10 lower than the cross over frequency of the compensated system.
With the compensator inserted and compensator amplifier gain set to unity, step response Fig. I(b) of Appendix-I gives

M_P = 25 %
t_p = 3 m sec.
e_{SS} = 3 %

It may be observed that compensation network has decreased the overshoot (smaller M_P) of the system while closely satisfying the e_{SS} requirements. Actual measurement of the phase margin would confirm that compensation has brought it to 40° (approx.).

(d) Design*: *(Lead network)

Continuing with the same design requirement as before, viz.

e_{SS} = 0.05 (5%)
Phase margin = 40°

We now proceed with the lead network design. To satisfy the steady state specifications, the open loop gain will need to be set to 5 as in the lag network example, leading to M_P = 48 %, e_{SS} = 5 %.

The gain setting of 5 must not be changed throughout this experiment.

The Phase Margin from Fig. 7 is read as 24°. Following the steps outlined in sec. 4.3,

Phase lead needed (ϕ_m) = 40° - 24° + 10° = 26°

$$\alpha = \frac{1-\sin 26}{1+\sin 26} = 0.39, \quad \frac{1}{\alpha} = 2.56$$

$$10 \log \alpha = -4.08 \text{ dB}$$

ω_m (from Fig. 7) = $2\pi 420$ rad/sec = 2638.93 rad/sec

$$\frac{1}{T} = \sqrt{\alpha\, \omega_m} = 1648.0, \text{ and } \frac{1}{\alpha T} = \frac{\omega_m}{\sqrt{\alpha}} = 4225.6$$

Time Constant T = R_1 C

Choosing C = 0.01 μF, the resistances are computed as

$$R_1 = \frac{T}{C} = 60.67 \text{ k}\Omega \approx 62 \text{ k}\Omega$$

$$R_2 = \frac{\alpha R_1}{(1-\alpha)} = 38.78 \text{ k}\Omega \approx 39 \text{ k}\Omega$$

With the compensator inserted and its gain set to 1/α = 2.5, the response of the closed loop system {Fig. I(c) of Appendix-I} yields:

M_P = 25 %
t_p = 1.3 m sec
e_{SS} = 3 %

An improved performance of the compensated system is obvious. Further, determination of the closed loop frequency response would show a phase margin of approximately 40°.

Note: All the measurements in this experiment are carried out on a CRO and therefore these may be accurate within a tolerance of about ± 5%. Further errors are caused by the non zero bias current requirement of the operational amplifiers used. For better results it is suggested that the gain settings of the system and compensation amplifier be actually measured. It is also recommended that R_1 and R_2 may not exceed 100 kΩ approximately and only polyester capacitors be used for C in the compensation network.

- *The traces shown in Fig. I(a-c) in Appendix-I are obtained with a Tektronics Digital Storage Oscilloscope, Type TDS 210*

6. REFERENCES

[1] K. Ogata, ‘Modern Control Engineering’, Prentice Hall of India Pvt. Ltd.

[2] B.C. Kuo, ‘Automatic Control Systems’ Prentice Hall of India Pvt. Ltd.

[3] Nagrath, I.J. and M. Gopal, ‘Control System Engineering’, Wiley Eastern Limited, 1975

APPENDIX – I

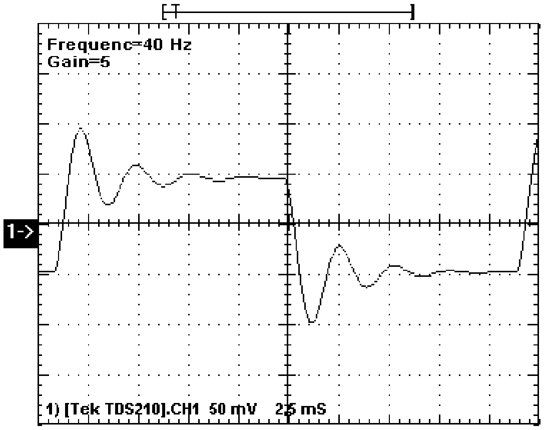


Fig. I. (a). Uncompensated System

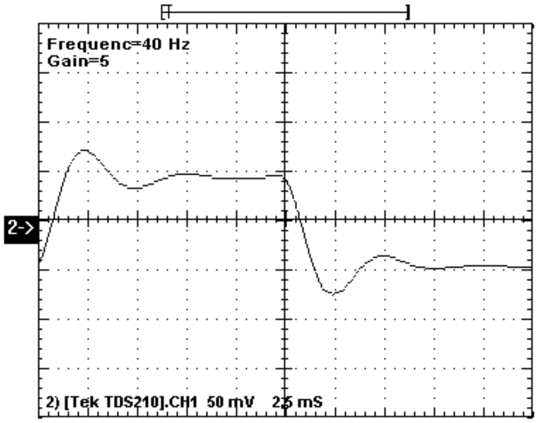


Fig. I. (b) Lag Network

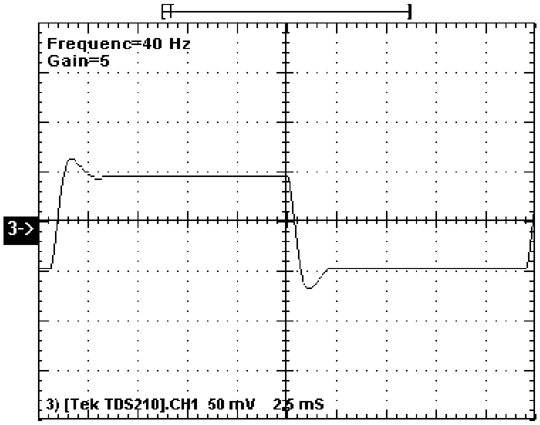


Fig. I. (c) Lead Network

Laboratory Manual

for

Control System Lab

Prepared by

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Electrical and Instrumentation Engineering Department

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D.C. SPEED CONTROL DCS-01

1. OBJECT

To study the performance characteristics of a d.c. motor speed control system.

2. EQUIPMENT DESCRIPTION

Speed control is a very common requirement in many industrial applications such as rolling mills, spinning mills, paper factories etc. The present unit is a low power d.c. motor speed control system designed as a laboratory experiment. The various components and subsystems have been carefully integrated, and the experiments are designed to illustrate the important performance characteristics in a simple way. Figure 1 shows a schematic of the system, different blocks and parts of which are described below :

(a) ***D.C. Motor*** : The 12 volt permanent magnet d.c. motor used in the system has the following specifications :

Rated voltage	:	12 volt d.c.
Rated current	:	200 mA at no load, 290 mA at full load
Torque	:	50 gm-cm
Maximum speed	:	3000 rpm

A slotted aluminium disk is mounted on the motor shaft which generates signals for speed measurement. Also, an adjustable eddy current brake is provided to enable the study of the effects of external disturbance on the system performance.

(b) ***Speed measurement*** : The slotted disk attached to the motor shaft generates 12 pulses for every revolution of the shaft through optical interruptions. After passing through signal conditioning and frequency scaling circuits, these pulses are then fed to a built-in frequency counter to display the shaft speed directly in rpm.

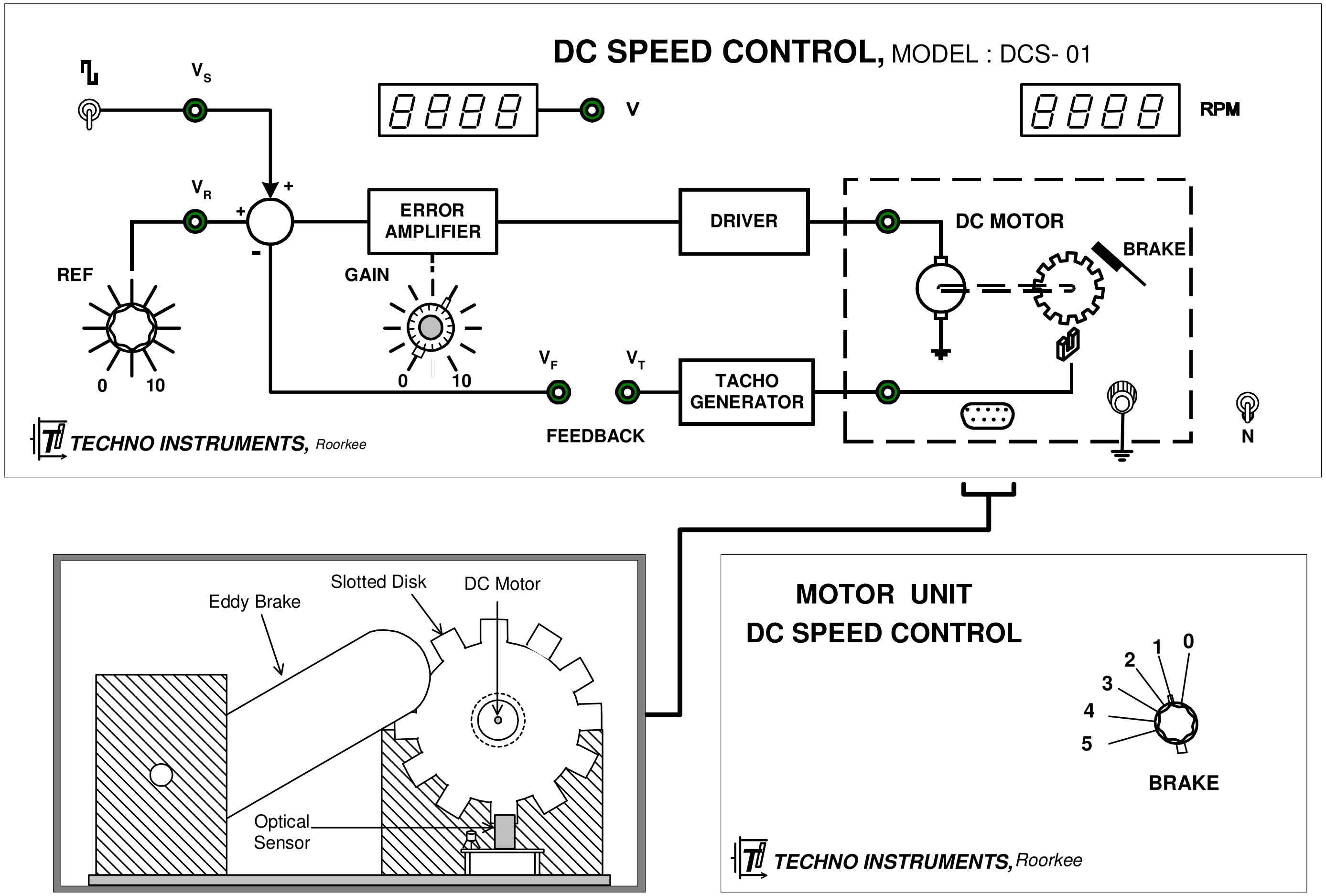
(c) ***Tachogenerator*** : A d.c. signal proportional to the shaft speed is obtained from an 'electronic tachogenerator' - a frequency to voltage converter circuit. The signal is brought to a suitable level by signal conditioning to yield a tacho constant of about 0.5 V/1000 rpm.

(d) ***Error Detector and Forward Gain*** : The speed signal obtained from the tachogenerator is compared with the reference (corresponding to a set speed) to obtain an error signal. The error is amplified by a calibrated variable gain amplifier (0-100) and then fed to the driver circuit.

(e) ***Driver Circuit*** : The driver circuit is designed to deliver the necessary power to operate the motor. It is a unity gain power amplifier and has all the necessary protection circuits

(f) ***Power and Signal Sources*** : A number of IC regulated supplies feed the electronic circuits, reference potentiometer, DVM, speed displays and the motor. Also, a square wave oscillator of 1 Hz (approx.) is included for time constant studies.

(g) ***DVM*** : A 19.99 Volt full-scale-deflection DVM mounted on the panel is available for the measurement of various signals. One terminal of the DVM is internally connected to ground.



Side View

Front View

Panel drawing DC Speed Control, Model DCS-01

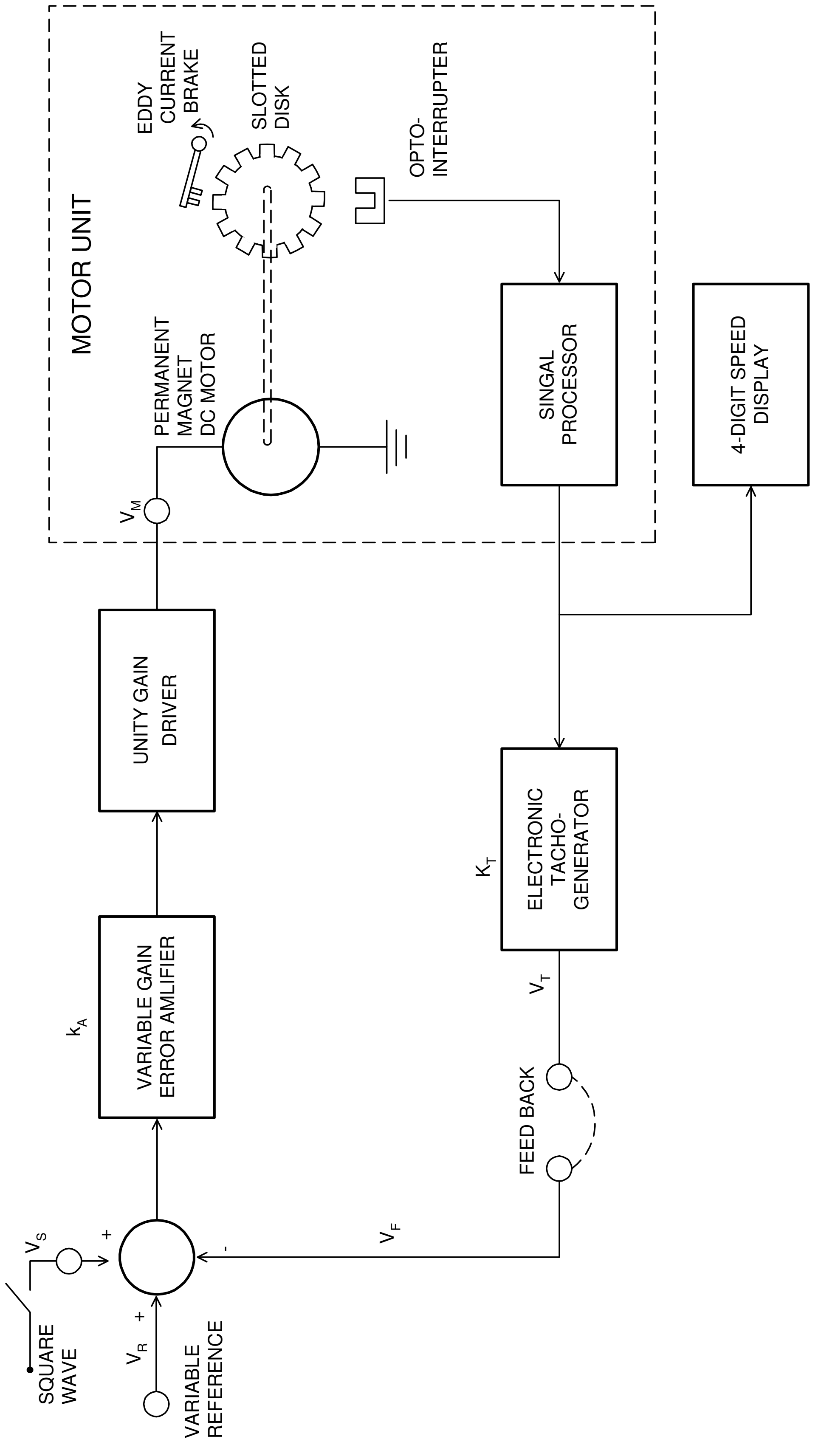


Fig.1 System schematic

3. BACKGROUND SUMMARY

A basic block diagram of the d.c. motor speed control system is shown in Fig. 2. In order to evaluate the system performance, it is necessary to compute the overall transfer function in terms of the transfer functions of the different blocks. To start with, the transfer function of an armature controlled d.c. motor of Fig. 3 may be derived as [1, page nos. 30-32],

$$\frac{\theta(s)}{V(s)} = \frac{K_M}{s(sT+1)}$$

where K_M is motor gain constant, and T is the mechanical time constant. Note that a permanent magnet d.c. motor should behave similar to a shunt motor with constant field excitation. Considering motor speed ω rad/sec (=dθ/dt) as the output variable, the forward path transfer function may be written as,

$$G(s) = \frac{\omega(s)}{V_E(s)} = K_A . \frac{K_M}{(sT+1)} \qquad (1)$$

where K_A is the gain of amplifier. Again, the tachogenerator transfer function (or gain) may be written as,

$$H(s) = \frac{V_T(s)}{\omega(s)} = K_T$$

This yields the closed loop transfer function of the complete system as

$$\frac{\omega(s)}{V_R(s)} = \frac{K_A K_M}{sT + K_A K_M K_T + 1} = \frac{\dfrac{K_A K_M}{K_A K_M K_T + 1}}{s\left[\dfrac{T}{K_A K_M K_T + 1}\right] + 1} \qquad (2)$$

In Eq. 2, the transfer function of the closed loop system is seen to be a first order type-0 function. Its transient and steady state response to step input may be easily studied as described below.

3.1 Steady State Error

Defining 'positional error coefficient', K_p as,

$$K_p = \lim_{s\to 0} G(s).H(s) = K_A.K_M.K_T,$$

the steady state error, e_{ss} , to step input R u(t), is given by,

$$\lim_{s\to\infty}(V_R - V_T) = \lim_{s\to\infty}(V_R - V_F) = e_{ss} = \frac{R}{1+K_P} = \frac{R}{1+K_A K_M K_T} \qquad (3)$$

- The steady state error may be determined from a measurement of V_R and V_F, and
- The steady state error is expected to decrease as K_A is increased.

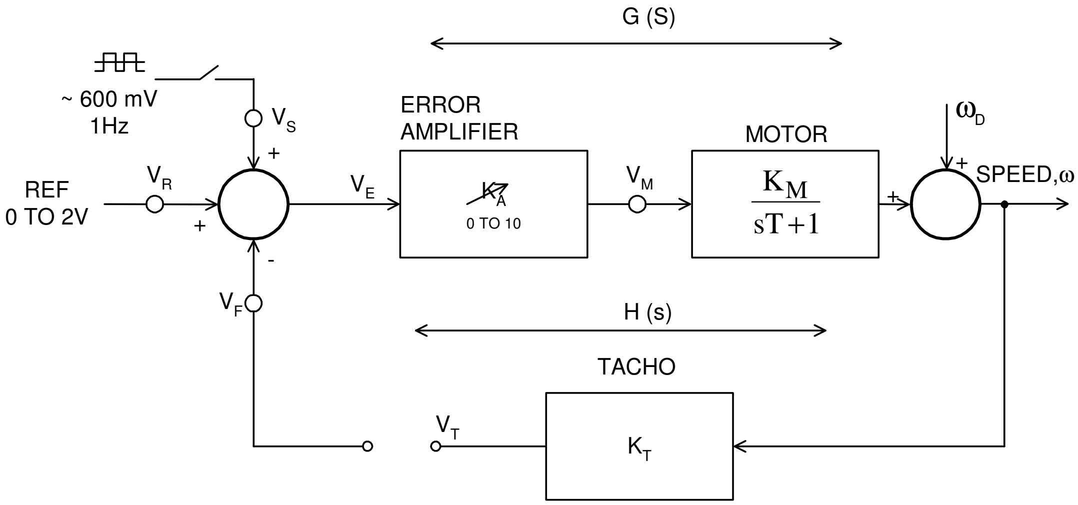


Fig.2 Block diagram

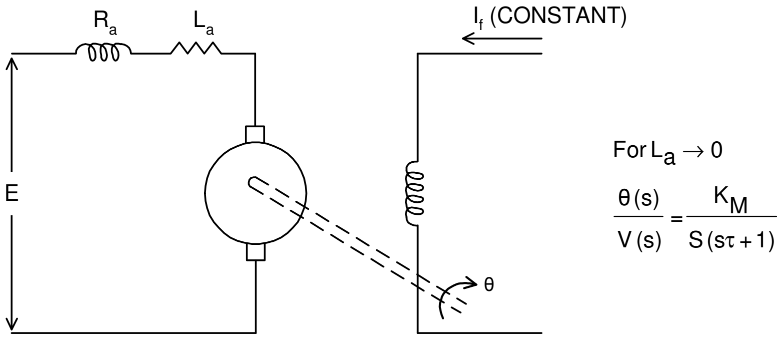


Fig. 3 Armature controlled DC motor

3.2 Transient Response

For a step input V_R (s) = R/s, Eq. 2 yields

$$\omega(s) = \frac{R}{s} \cdot \frac{K_A K_M / T}{s + (K_A K_M K_T + 1)/T}$$

Taking inverse Laplace transform

$$\omega(t) = \frac{R.K_A K_M}{K_A K_M K_T + 1} \left(1 - e^{-\frac{K_A K_M K_T + 1}{T}.t} \right) \tag{4}$$

where effective time constant T_{eff} may be defined as,

$$T_{eff} = T/(K_A K_M K_T + 1) \tag{5}$$

The transient response has an exponential character similar to a capacitor charging through a resistor. Further, the effective time constant T_{eff} decreases with increasing K_A making the motor response faster.

The effective time constant may be determined from a recording of the step response using either a pen recorder or a storage CRO. The step response for various values of K_A obtained through a storage oscilloscope, Tektronics, Model: TDS-210 is shown in Fig. 4 (a). It may further be observed that for large gains (≥60) the speed of response becomes constant due to saturation of amplifier and/or motor. The initial portion of the response is therefore a straight line. Time constant may also be computed using an ordinary CRO as explained next.

Consider a general first order, type-0 transfer function of the form

$$\frac{C(s)}{R(s)} = \frac{K}{s\tau + 1}$$

which may represent both open loop and closed loop speed control systems defined by Eqs. (1) and (2). Its response to a step input, R u(t) may be seen to be

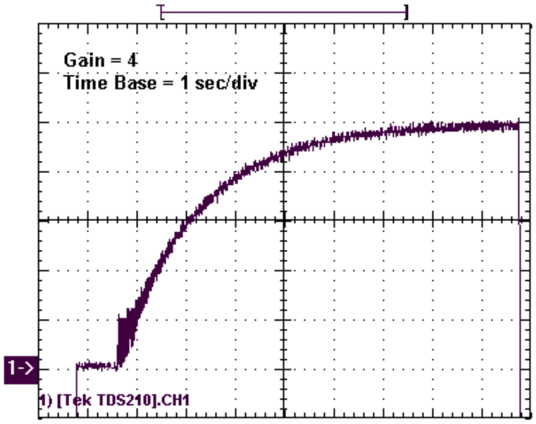
$$c(t) = R.K.(1-\exp(-t/\tau))$$

For a square wave of p-p value of R as input, refering to Fig. 4 (b) it is easy to see that

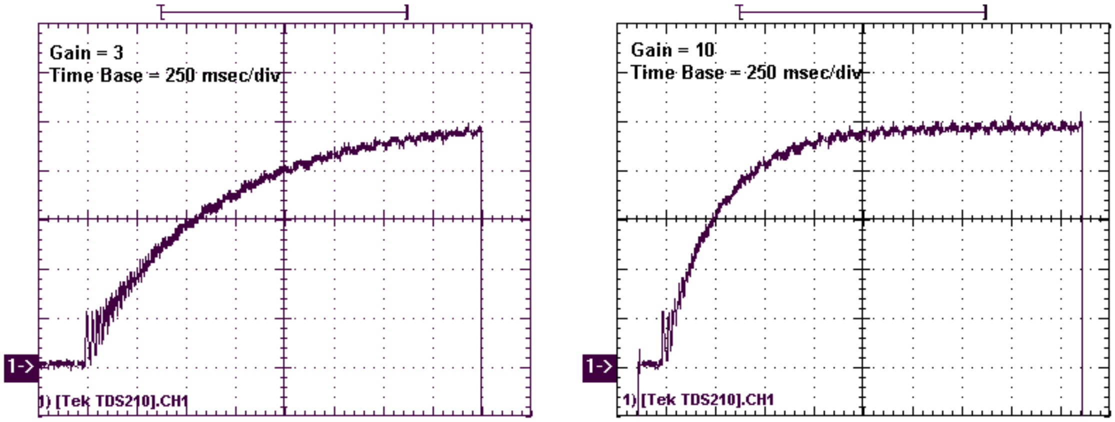
$$\tau = \frac{1}{2f} \cdot \frac{1}{\ln\left(1 - \frac{C(p-p)}{R(p-p)} \cdot \frac{1}{K}\right)} \tag{6}$$

where f is the frequency of the squarewave. The above equation suggests a method for computing the motor time constant T.

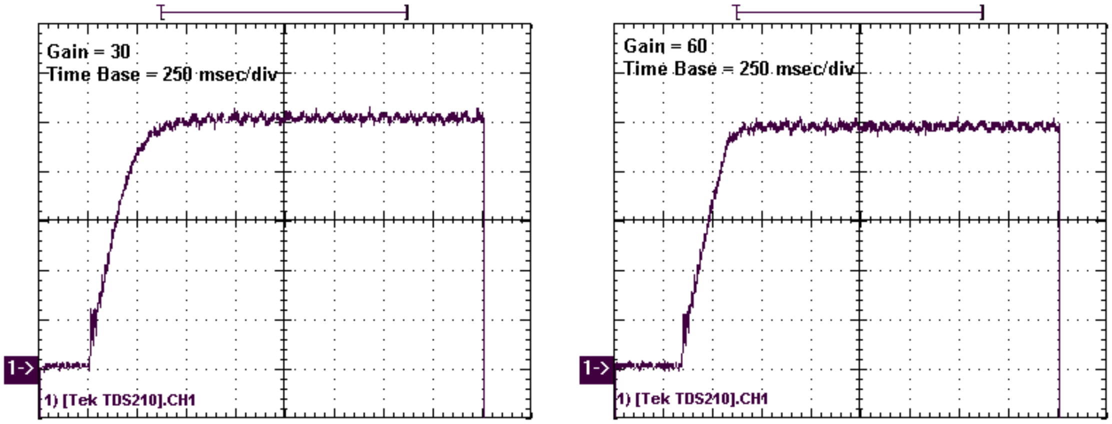
In the present electromechanical system, however, it is the shaft speed ω which will go through a triangular wave type of variation in response to a square wave excitation, i.e. C=ω(p-p). Since we are measuring the shaft speed using tachogenerator, ω(p-p), given by V_T(p-p)/K_T, and V_M(p-p), the motor input, the time constant may be found from the equation,



Open Loop



Closed Loop - High Gain **Closed Loop - Very High Gain**



Closed Loop - High Gain **Closed Loop - Very High Gain**

Fig 4 (a) Step Response Recording

$$T = \frac{1}{2f} \cdot \frac{1}{\ln\left(1 - \frac{V_T\,(p-p)}{V_M\,(p-p)}\right)} \cdot \frac{1}{K_M\,K_T}\ , \qquad \text{for open loop system} \qquad 7\,(a)$$

Where V_M (p-p) = K_A-V_S (p-p)

and

$$T = \frac{1}{2f} \cdot \frac{1}{\left[1 - \frac{V_T(p-p)}{V_M(p-p)} \cdot \frac{1}{K_M K_T}\right]}, \qquad \text{for closed loop system} \qquad 7\,(b)$$

where K includes only $K_M K_T$ product from Eq. (1).

3.3 Disturbance Rejection

One of the important features of a feedback control system is its ability to reduce the effect of external disturbances. From Fig. 2, the disturbance transfer function for V_R=0, may be written as

$$\frac{\omega(s)}{\omega_D(s)} = \frac{1}{1 + G(s)H(s)} = \frac{sT + 1}{sT + 1 + K_A K_M K_T}$$

For a unit step disturbance, $\omega_D(s)=\Omega_D/s$, the steady state output speed is given by

$$\omega_{ss} = \frac{\Omega_D}{K_A K_M K_T + 1} \qquad (8)$$

Thus, the steady state speed change caused by an external disturbance should reduce as the gain K_A is increased. Also, the performance should be much superior to the open loop case, i.e. with feedback disconnected (K_T=0)

In the experimental unit, the external disturbance is created by an eddy current brake. The pole pieces of a permanent magnet are inserted to varying depths into the rotating aluminium disk. The eddy currents induced in the disk result in power loss and thereby load the motor.

4. EXPERIMENTAL WORK

The experiments suggested in this section start with a study of the open loop system and its subsystems. This is followed by the performance evaluation of the closed loop system for various operating conditions like forward path gain and disturbance.

4.1 Subsystem Performance

Various subsystem blocks are shown in Fig. 1 and 2. The characteristics of motor, tachogenerator and square wave source are determined first. The FEEDBACK terminals are left open during this experiment. **Note that K_A may be varied from 0 to 100 using a 10-turn potentiometer. Thus one turn of the potentiometer corresponds to gain variation from 0 to 10**.

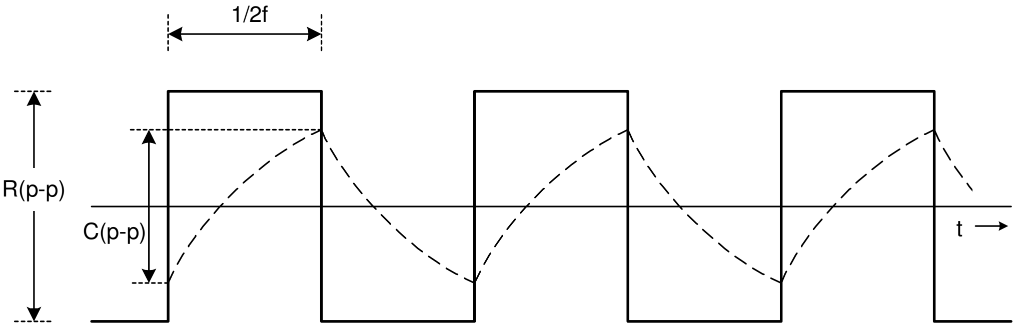


Fig 4(b) Input-output waveform with square wave excitation

(a) Signal and reference

- Set K_A=0. Connect DVM to measure the range of variation of reference voltage V_R.
- Switch ON the square wave signal V_S and measure its amplitude and frequency using a calibrated CRO. The frequency of this signal is about 1Hz., which makes the CRO display very inconvenient for measurements. It is suggested that the amplitude may be measured with time-base switched OFF, and for frequency, simply count the number of pulses (as seen on CRO screen), in say 60 seconds, using a watch.

(b) Motor and Tachogenerator

- Set V_R=1 Volt and K_A=3. The motor may be running at a low speed. Record speed N in rpm, and the Tachogenerator output V_T.
- Repeat with V_R=1 and K_A=4, 5, ..., 10, and tabulate measured motor voltage V_M (=$V_R K_A$), steady state motor speed N in rpm (or ω_{ss}=Nx2π/60 in radians/sec.) and tachogenerator output V_T.
- Plot N vs. V_M, and V_T vs. N. Obtain K_M and K_T from the linear region of the curves (see Fig. 5).

$$\text{Motor gain constant, } K_M = \frac{\text{shaft speed in rad/ sec, } \omega_{ss}}{\text{Motor voltage, } V_M}\text{, and}$$

$$\text{Tachogenerator gain, } K_T = \frac{V_T}{\omega_{ss}} \quad \frac{\text{volt - sec}}{\text{rad}}$$

- To calculate motor time constant, with square wave signal V_S ON, set V_R and K_A so that the peak-to-peak variation of V_M lies between 3-8V. This would ensure a reasonably linear operation of the motor. Use Eq.(7) to calculate the motor time constant T.

  (Caution: The CRO must be kept in 'dc-input' mode for this measurement.)
- Obtain the motor transfer function using

$$G(s) = \frac{K_M}{sT + 1}$$

(c) Disturbance

- Set K_A=5 and adjust the reference V_R to get a speed reading close to 1200 rpm. The brake setting should be at 0 i.e. no braking.
- Record and tabulate the motor speed variation for different settings of the eddy current brake.
- Calculate percentage decrease in speed at each setting of the brake, starting from no braking.

4.2 Closed Loop Performance

Performance of the closed loop system is evaluated in terms of steady state error and disturbance rejection as functions of forward gain. The FEEDBACK terminals are connected together for this experiment.

(a) Steady state error

- Set V_R=1 volt and K_A=5. The motor may be running at a low speed. Measure and record speed N in rpm, tachogenerator voltage V_T, and the steady state error e_{ss} (=V_R-V_T).
- Repeat above for K_A = 5, 10, 15, 20, …….100
- Compare in each case the value of steady state error computed from Eq. (3) i.e.

$$e_{ss} = \frac{1}{1+K_A K_M K_T}$$

- Comment on the results.

(b) Transient Performance

- Set V_R=0.5 Volt and K_A=5. Switch ON the square wave signal and measure peak-to-peak amplitudes of V_S and V_T. Use Eq. (7) to calculate system time constant T_{eff} with R=V_S(p-p) and C=V_T(p-p). The value of K may be obtained from (2) as

$$K = \frac{K_A K_M}{K_A K_M K_T + 1}$$

(Note: Although the method suggested above is theoretically valid, in the experimental unit the results are likely to be erroneous. As the closed loop system encounters a step input, the motor is driven harder (increased input) so as to force it to respond faster. The current limitation and saturation in the amplifier does not permit this in a linear fashion. However for small values of K, say less than 5, the results are reasonable.)

(c) Disturbance Rejection

- With K_A=5, FEEDBACK terminals shorted and the brake setting at 0, adjust reference V_R to get a speed close to 1200 rpm.
- Record and tabulate the variation in speed for different settings of the eddy current brake. Calculate percentage decrease in speed at each setting of the brake.
- Repeat above for K_A=10, 50, 100.
- Compare the percentage decrease in speed at various brake settings for open loop, closed loop with K_A=5, and closed loop with K_A=10. Comment on the results.

5. RESULTS

Typical results obtained on an experimental unit are given below for guidance.

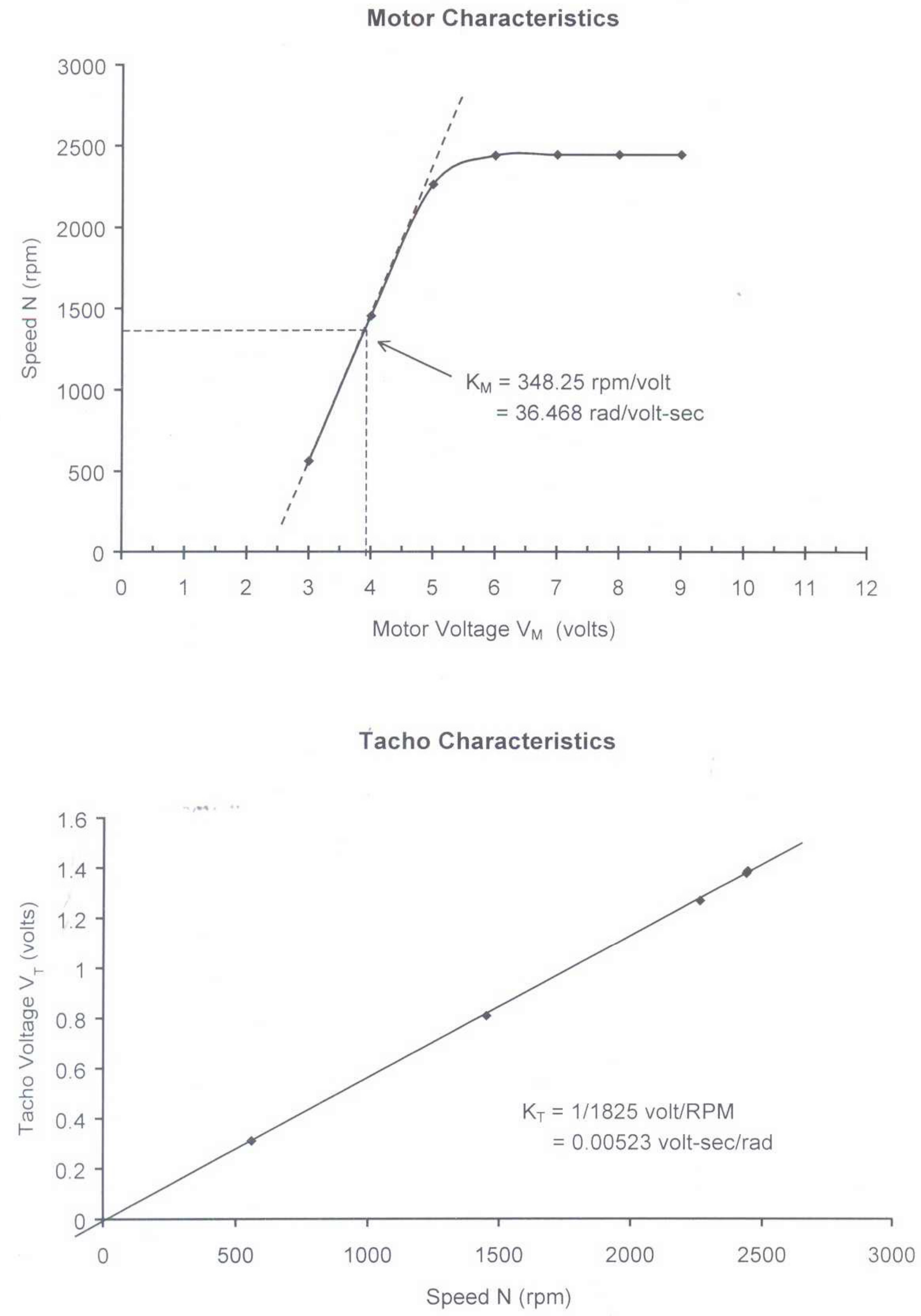


Fig 5: Typical Experimental Characteristics of the Motor and the Tachogenerator

(a) Motor and Tachogenerator Characteristics

V_R=1 volt

S.No.	K_A Setting	N rpm	V_T Volt	V_M Volt	Experimental $K_A=V_M/V_R$
1	3	560	0.31	3.01	3.01
2	4	1453	0.81	4.19	4.19
3	5	2264	1.27	5.45	5.45
4	6	2440	1.38	6.46	6.46
5	7	2445	1.39	7.44	7.44
6	8	2445	1.39	8.55	8.55
7	9	2445	1.39	9.59	9.59

Graphs of N vs. V_M, and V_T vs. N are shown in Fig. 5. From the linear region, K_M and K_T are obtained as, K_M=36.468, K_T=0.00523, and K_MK_T=0.19

Time constant was obtained as

$$T = \frac{1}{2f}\cdot\frac{1}{\ln\left[1-\frac{V_T(p-p)}{V_M(p-p)}\cdot\frac{1}{K_MK_T}\right]} = \frac{1}{2\times 0.82\ln\left[1-\frac{0.04}{0.8}\cdot\frac{1}{0.19}\right]} = 1.997 \text{ sec.}$$

Where $f = 0.82$Hz., $V_M = 0.8$V(p-p), $V_T = 40$mV (p-p)

and the motor transfer function as

$$G(s) = \frac{K_M}{sT+1} = \frac{36.468}{1.997s+1}$$

Note that the motor speed-voltage characteristics is rather non-linear. This is because the motor fails to start at very low voltages and at higher voltages its speed saturates due to internal speed limiter.

(b) Closed loop performance

(i) Steady state error

V_R=1 volt

S.No.	K_A	N rpm	V_T Volt	$e_{ss}=(V_R-V_T)$ Experimental	$e_{ss}=1/(1+K_AK_MK_T)$ Theoretical
1	5	648	0.37	0.63	0.516
2	10	1083	0.62	0.38	0.348
3	15	1276	0.73	0.27	0.262
4	20	1377	0.79	0.21	0.210
5	25	1442	0.82	0.18	0.175
6	50	1581	0.90	0.10	0.096
7	75	1635	0.93	0.07	0.066
8	100	1656	0.94	0.06	0.0506

Observe that the numerical values of theoretical and experimentally obtained steady state error do not match, though the pattern of variation is same i.e it decreases with increase in forward gain. The mismatch is due to the fact that the motor gain constant K_M does not remain constant due to non-linearity of the motor.

(ii) System time constant

$V_R = 0.5$ V d.c.

$f = 0.82$ Hz

$K_A = 5$, $V_P = 480$ mV (p-p), $V_T = 120$ mV (p-p)

From equation 7(b)

$$K_{eff} = \frac{K_A\, K_M\, K_T}{1 + K_A\, K_M\, K_T} = \frac{5 \times 0.19}{1 + 5 \times 0.19} = 0.487, \quad \text{and}$$

$$T_{eff} = \frac{1}{2 \times 0.82} \cdot \frac{1}{\ln\left(1 - \frac{1}{0.487} \cdot \frac{120}{480}\right)} = 846 \text{ msec}$$

(iii) Disturbance rejection

Speed = 1200 rpm (approx.)

The table below shows the variation of speed under various conditions of feedback and thus illustrates the effectiveness of speed control system in rejecting disturbance.

Brake Setting	0	1	2	3	4	5
Open Loop Speed, rpm	1204	1200	1136	812	596	563
Closed Loop (K_A=5)	1205	1203	1182	1023	910	886
Closed Loop (K_A=10)	1208	1205	1189	1107	1010	991
Closed Loop (K_A=50)	1186	1186	1183	1156	1134	1128
Closed Loop (K_A=100)	1196	1195	1192	1178	1167	1164

6. REFERENCES

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Laboratory Manual

for

Control System Lab

Prepared by

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D.C. MOTOR STUDY DCM-01

1. OBJECTIVE

To study the torque-speed characteristics and determine the transfer function of a d.c. motor.

2. EQUIPMENT DESCRIPTION

D.C. motors are the most commonly used actuators in electro-mechanical control systems or servomechanisms. Compared to actuators like 2-phase a.c. motor and stepper motor, the d.c. motor has the advantage of higher torque and simpler driving circuit. However the presence of a commutator and a set of brushes with the problems of sparking make the d.c. motor somewhat less durable. This of course is not true for a present day well designed d.c. servomotor.

The study of the dynamic characteristics of the d.c. motor is important because the overall performance of the control system depends on it. A standard analysis procedure is to model the various subsystems and then combine these to develop the model of the overall system.

This experiment is designed to obtain the torque-speed characteristics, compute the various parameter and finally determine the transfer function of a d.c. motor.

The various sections of the unit are described below in some detail.

(a) ***Mechanical Section*** **:** It comprises of the experimental permanent magnet d.c. motor (approx. 8W) coupled to a small d.c. generator (approx. 2W), which serves the twin purposes of,

- electrical loading of the motor, and
- transient response signal pick-up.

Further, a slotted disk mounted on the common shaft produces 6 pulses per revolution through an opto- interrupter, which is used in a 4-digit speed display in r.p.m.

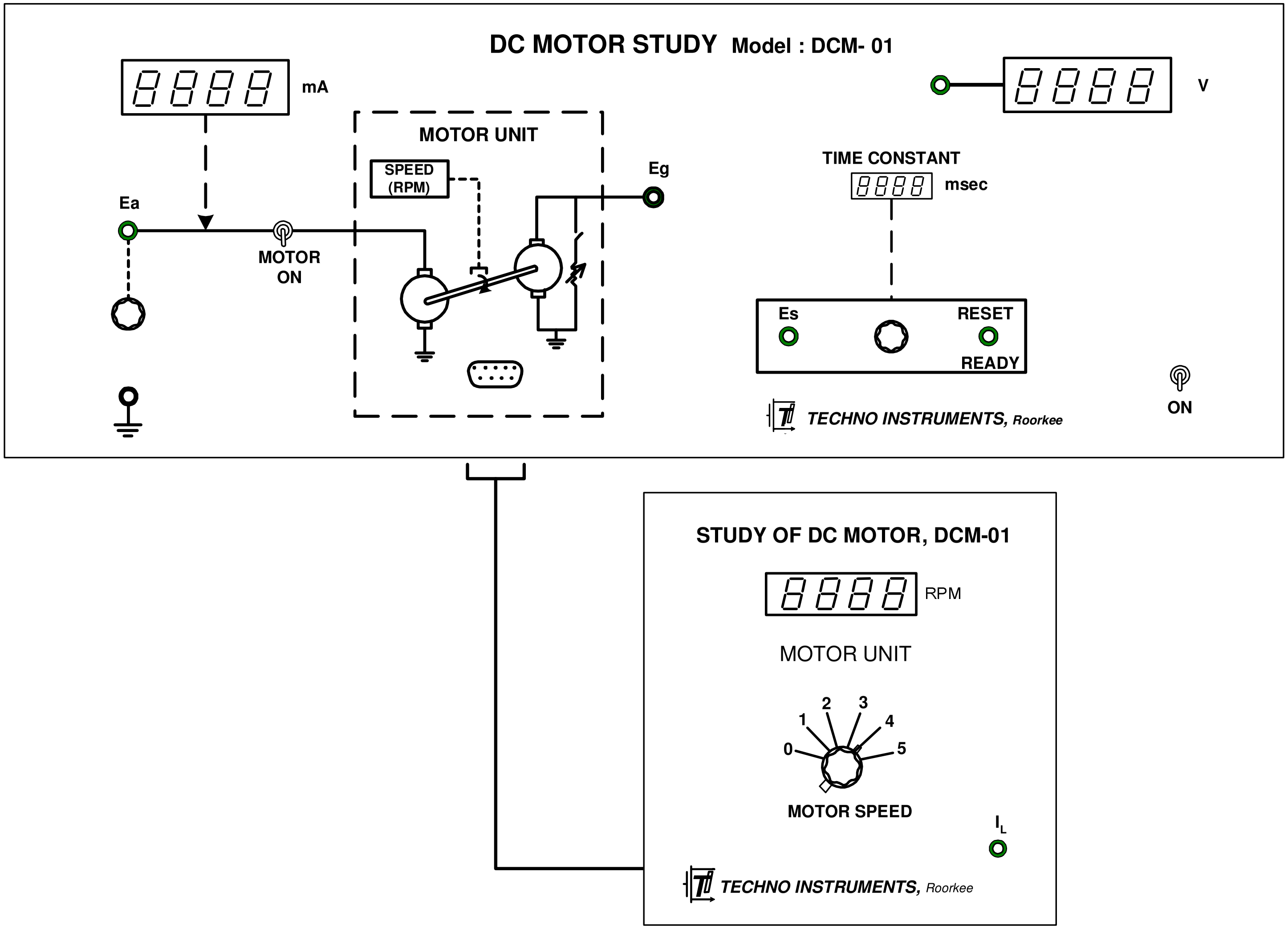
The specifications of the main experimental d.c. motor are:

- Operating Voltage :12Vdc
- No Load Current: 0.09A
- Full Load Current: 1.0A
- Torque: 30mN-m/ 300g-cm

(b) ***Motor Power Supply*** : The operating voltage of the motor is 12 volt d.c. while the current, depending on loading, is around 120-650 mA. A built-in variable voltage source (2-14Vd.c.) provides this power and two 3½ digit DPMs are available to monitor the armature voltage and armature current of the motor.

(c) ***Transient Response Timing Section*** : When the power is suddenly switched ON, the motor speed increases gradually and finally reaches a steady value. This process takes a few tens of milliseconds and is therefore too slow for a CRO display with repeated ON/OFF of the motor. Although a storage CRO could be used to freeze the transient response and compute the time constant, an alternative using digital circuits is provided in the unit. A 3-digit time count display enables the user to measure the time constant without an expensive storage CRO. This is explained in section 3.3.

(d) ***Power Supplies*** : All the circuits are powered through built-in I.C. regulated power supply of appropriate capacities.



Panel drawing DC Motor Study, DCM-01

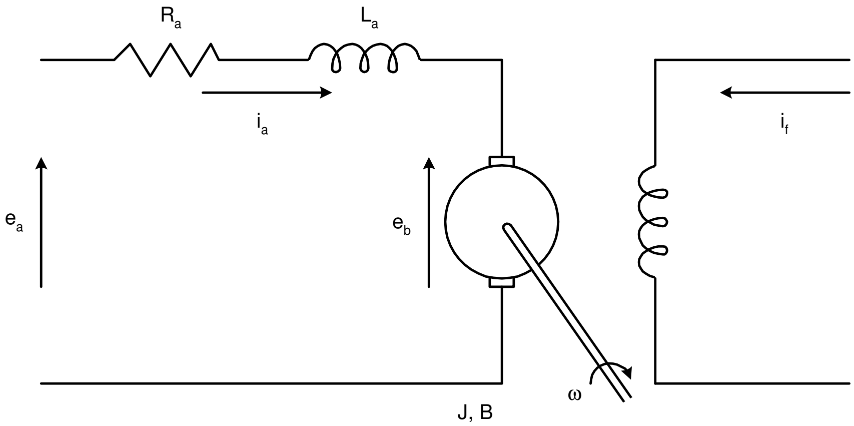


Fig. 1 Schematic diagram

3. BACKGROUND SUMMARY

3.1 D.C. Motor Model [1]

The schematic diagram of a d.c. motor is shown in Fig.1 wherein the following notations are used

e_a : armature voltage (volts)
i_a : armature current (amp.)
R_a : armature resistance (ohms)
L_a : armature inductance (henrys)
e_b : back emf (volts)
i_f : field current (amp.)
T_M : motor torque (newton-m)
T_L : load torque (newton-m)
ω : angular velocity (rad/sec)
J : moment of inertia of the rotor including external loading if any (newton-m/rad/sec^2)
B : viscous friction coefficient including external loading if any (newton-m/rad/sec)

Upper case notations E_a, I_a, E_b, I_f are used for steady state values of the respective variables e_a, i_a, e_b and i_f

In the present set-up a permanent magnet d.c. motor is used, the field winding is thus absent and the air gap flux is constant. The input drive may therefore be applied to the armature only, that is, only armature controlled operation is possible.

The mathematical equations in this operating mode are,

$T_M = K_T i_a$; K_T : torque constant

$e_b = K_b\omega$; K_b : back emf constant

$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$; armature circuit model

$J \frac{d\omega}{dt} + B\omega + T_L = T_M$; mechanical model

Taking Laplace Transform and rearranging the terms,

$$\frac{\omega(s)}{E_a(s)} = \frac{K_T}{(sL_a + R_a)(sJ + B) + K_T K_b}$$

Assuming the inductance of the armature circuit to be very small*, the motor transfer function may be written as,

---

* For the motor used $R_a \cong 4\Omega$ and $L_a \cong 2.9$mH. Thus even operating at 10 Hz, $\omega L_a = 0.182$, which can be neglected in comparison to R_a.

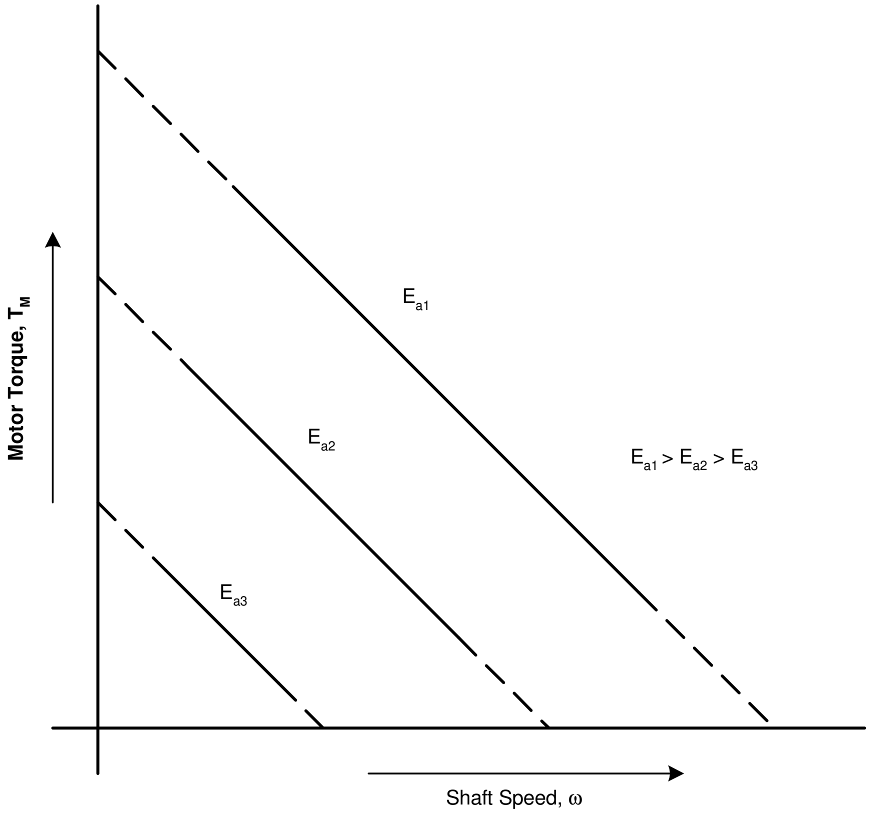


Fig. 2 Typical torque - speed curve

$$G_M(s) \triangleq \frac{\omega(s)}{E_a(s)} \approx \frac{K_T/R_a}{Js + B + \frac{K_T K_b}{R_a}} = \frac{K_M}{s\tau_m + 1} \qquad \text{... (1)}$$

where,

$$K_M = \frac{K_T}{R_a B + K_T K_b} \quad : \text{ Motor gain constant}$$

$$\tau_m = \frac{R_a J}{R_a B + K_T K_b} \quad : \text{ Motor time constant}$$

The armature controlled motor therefore has a first order type-0 transfer function as shown in eqn. (1) and the two constant K_M and τ_m depend upon motor parameters and external loading, if any.

If the transfer function is defined with respect to shaft position (θ), rather than velocity ($\omega = d\theta/dt$), the transfer function may be written as,

$$G'_M(s) \triangleq \frac{\theta(s)}{E(s)} = \frac{K_M}{s(s\tau_m + 1)}$$

This form would be appropriate if one is interested in shaft position as the output, e.g., in a position control system like the D.C. Position Control, DCP-01 manufactured by M/s Techno Instruments, Roorkee.

3.2 Torque – Speed Curves

As a mechanical actuator the magnitude of the steady state torque produced by the motor with a given armature voltage is of interest to an user. With a simple rearrangement of terms the motor torque may be written as,

steady state armature current, $I_a = \frac{E_a - E_b}{R_a} = \frac{E_a}{R_a} - \frac{K_b \omega}{R_a}$

steady state torque generated, $T_M = K_T \; I_a = -\frac{K_T K_b}{R_a} \cdot \omega + \frac{K_T}{R_a} \cdot E_a$ ... (2)

Here T_M, E_a, E_b, I_a and ω are the steady state values of the motor torque, applied armature voltage, back emf, armature current and angular velocity of the shaft.

A typical plot of the above equation is shown in Fig. 2. This assumes a linear torque-speed behaviour as indicated in eqn. (2). A practical d.c. motor/d.c. servomotor will show nonlinearity to varying extent depending upon the design and manufacturing aspects. Since a linear model (transfer function) is used in the current experiment, salient features may be studied using any good quality d.c. motor. The plotting of Fig. 2 from experimental data is a little involved because of the difficulty in experimentally determining K_T. A relationship between K_b and K_T developed here makes the task simpler.

As the motor runs at constant speed,

Electrical power input, $P_{in} = E_a \times I_a$ watts

Power lost in $R_a = R_a \times I_a \times I_a$

Power available in the armature, $P_{arm} = (E_a - I_a R_a) I_a$

$= E_b \times I_a$

$= K_b \times \omega \times I_a$

Mechanical power developed, $P_{mech.} = T_M \times \omega$ newton-m rad/sec

$$= K_T \times I_a \times \omega$$

Assuming 100% conversion of power from electrical input to mechanical output, the above two expressions can be equated to get

$$K_b \left(\frac{\text{volts}}{\text{rad / sec}}\right) = K_T \left(\frac{\text{newton} - \text{m}}{\text{amp.}}\right)$$

Thus, the numerical values of K_T and K_b may be assumed to be identical.

The torque may then be expressed as,

$$T_M = -\frac{K_b^2}{R_a} \cdot \omega + \frac{K_b}{R_a} \cdot E_a \qquad \text{... (3)}$$

This equation may be obtained experimentally with ease since it is very simple to determine K_b.

When the motor is loaded, the speed decreases which reduces the back emf. This increases armature current i_a so that the motor develops more torque in order to supply the load. The operation for a constant voltage E_a is represented as in a straight line in Fig. 2.

At steady state (ω = constant) the load torque equation must read as

$$T_M = B\omega + T_L, \qquad T_L \; : \; \text{load torque} \qquad \text{... (4)}$$

In the experimental work T_L is increased in steps by loading the motor with the help of the coupled generator and the values of T_M and ω are recorded. While ω is computed from the speed N, in rpm, as displayed on the motor unit, the following expression is used to compute the motor torque T_M at a constant value of E_a,

$$T_M = K_T I_a = K_b I_a = \frac{E_b}{\omega} \cdot I_a = \frac{E_a - I_a R_a}{\omega} \cdot I_a \qquad \text{... (5)}$$

This is essentially same as (3) while avoiding the explicit computation of K_b.

Note that equations (3) and (4) both give the variation of T_M with ω for a constant armature voltage and are therefore basically the same, subject to the assumption that $K_T = K_b$. In the experiment however eqn. (4) is not used since it involves measurement of load torque T_L.

The value of B, coefficient of viscous friction, may be seen as the negative of the slope of torque speed curve eqn. (4) and K_b may be computed from the expression

$$K_b = \frac{E_b}{\omega} = \frac{E_a - I_a R_a}{\omega}$$

Two motor parameters, B and K_b, may therefore be determined from the Torque-Speed Characteristics obtained under steady state conditions or constant speed operation of the motor

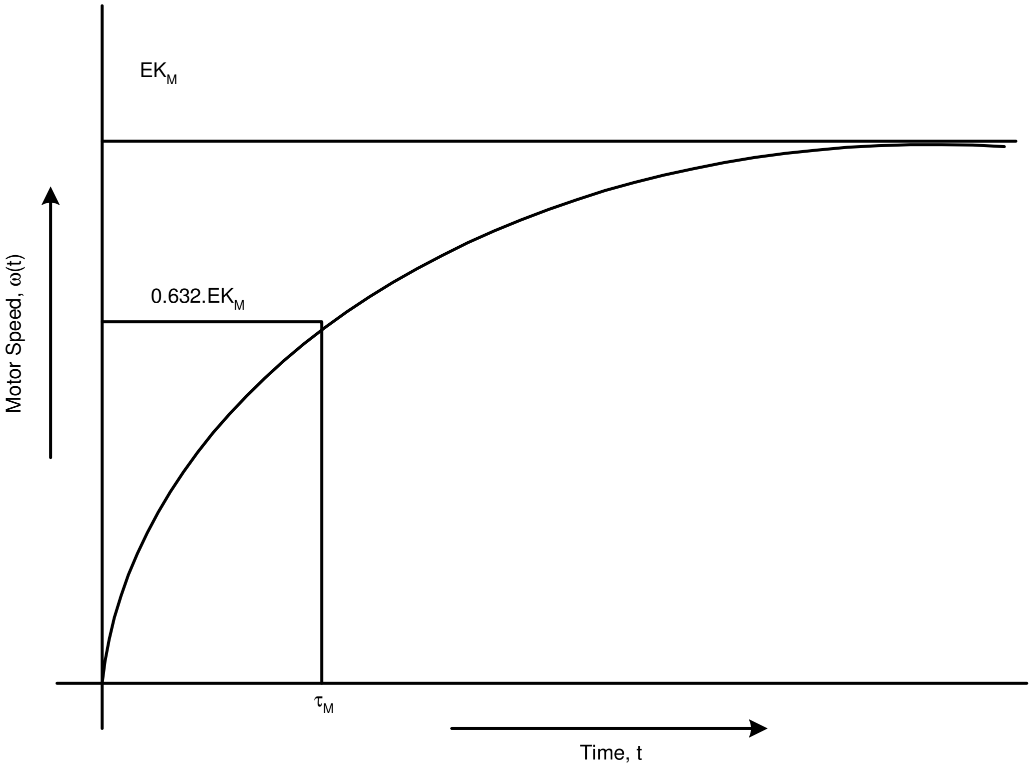


Fig. 3 Step response of the motor

3.3 Transient Response

The transfer function of the motor was obtained in eqn.(1) as,

$$G_M(s) = \frac{\omega(s)}{E_a(s)} = \frac{K_T/R_a}{Js + B + \frac{K_T K_b}{R_a}} = \frac{K_M}{s\tau_m + 1} \qquad \text{... (6)}$$

where,

$$K_M = \frac{K_T}{R_a B + K_T K_b} \quad : \text{ Motor gain constant}$$

$$\tau_m = \frac{R_a J}{R_a B + K_T K_b} \quad : \text{ Motor time constant}$$

In response to a step input, $e_a(t) = E.u(t)$, i.e. $E_a(s) = \frac{1}{s} \cdot E$, the motor speed will follow the expression,

$$\omega(t) = E \, . \, K_M \left(1 - e^{-\frac{t}{\tau_m}} \right), \text{ as shown in Fig. 3}$$

The step response is very similar to that of an RC circuit charging from a step voltage input. The parameters of interest, viz., EK_M and τ_m are indicated in Fig. 3. One can easily measure the steady state speed, N, and hence compute K_M,

$$\omega(t)\big|_{t\to\infty} = \omega_{ss} = E \, K_M$$

$$\text{or, } K_M = \frac{N}{E} \text{ rpm/volt } = \frac{\pi N}{30E_a} \quad \frac{\text{rad/sec}}{\text{volt}}$$

Measurement of τ_m, the time taken by the motor speed to rise from zero and attain 63.2% of ω_{ss}, is a little difficult because of the following facts:

(a) The motor being a mechanical system, takes a long time (approx. 500 msec.) to reach near $\omega_{ss.}$ If a steady trace on the CRO is to be displayed, the motor must be switched ON and OFF at around 1Hz. This is too low a frequency for convenient viewing.

(b) Even if the above scheme was possible, the time constants during ON and OFF would be different, since the back emf would be absent in the latter case (armature circuit is disconnected). The switching frequency will then need to be still lower.

It is of course possible to use a storage CRO or pen recorder to freeze the transient and make measurements at a later time. In the present unit, a class room experiment, a 3-digit timer has been provided which gives the time elapsed in milli seconds between starting the motor and reaching a preset speed, which would be set to 63.2% of the final speed. This then directly gives the time constant value with a least count of 1 msec.

From the motor time constant τ_m obtained experimentally, the value of coefficient of inertia (J) may be computed using eqn. (1) as,

$$J = \tau_m \left(B + \frac{K_b^2}{R_a} \right) \qquad \ldots(7)$$

Also an explicit expression for the motor transfer function may than be written.

The transient response study gives the value of K_M and τ_m. All the motor parameters and its transfer function may then be calculated using these and the parameters obtained in section 3.2.

4. EXPERIMENTAL WORK

This section deals with the details of the suggested experimental work, typical results and calculations. It must be emphasized here that the typical results given below are based on the measurements made on an unit randomly picked up from our assembly line. Although these results are indicative of the general characteristics, they cannot be expected to be exactly duplicated on other units.

4.1 Armature Circuit Parameters

R_a and L_a are the two parameters in the armature circuit, which may be measured by any standard method. Although R_a is required for calculations in the next section, L_a has been neglected as explained in sec 3.1.

4.2 Motor and Generator Characteristics

At no load (load step at 0) the motor is supplied with varying armature voltages, E_a = 3, 4, 5, …12. For each E_a, the motor current I_a, speed N, and generator voltage E_g are recorded. Straight line approximation of the E_a vs. speed and E_g vs. speed yield the motor and generator constants K_M (rpm/volt) and K_G (volts/rpm). Referring to the panel diagram on page 11, the following steps are suggested :

- Set 'MOTOR' switch to 'ON'. Set 'RESET' switch to 'RESET'. Set 'LOAD' switch to 0 position.
- Vary E_a in small steps and take readings as under (Table 1)

Table -1

S.No.	E_a, volts	I_a, amp.	N, rpm	E_g, volts
1.	3			
2.	4			
3.				
4.				

- Plot N vs. E_a and E_g vs. N. Obtain the slopes and compute K_M and K_G. {Ref. Fig. 5(a) and 5(b)}.

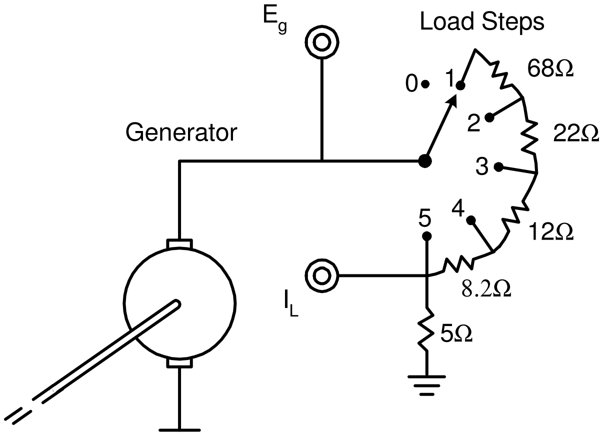


Fig. 4   Loading system Schematic

4.3 Torque Speed Characteristics

To obtain the torque-speed characteristics, the motor is supplied with a fixed armature voltage and its speed is recorded for varying external loading (Fig. 4). This loading is effected by electrically loading the coupled generator. Referring to the panel diagram on page 11, following steps are suggested:

- Set 'MOTOR' switch to 'OFF'. Set 'RESET' switch to 'RESET'. Set 'LOAD' switch to 0 position.
- Connect E_a to the voltmeter and set $E_a = 6V$
- Shift the 'MOTOR' switch to 'ON'. Measure armature input (E_a), motor current (I_a) and motor speed in rpm. Record the readings (at S. No. 1 in the Table-2)
- Set the 'LOAD' switch to 1, 2, …5 and take readings as above. (S.Nos. 2, 3, … in the Table 2)
- Complete the table below with the calculated values
  Motor Voltage $E_a = 6$ volts; $R_a = 4\Omega$

Table – 2

S.No.	Load Step	I_a, mA	N, rpm	$\omega = \frac{2\pi N}{60} = \frac{N\pi}{30}$ rad/sec.	$E_b = E_a \_ I_a R_a$ Volts	$K_b = \frac{E_b}{\omega}$	$T_M = K_b I_a$ newton-m
1.	0						
2.	1						
3.	2						
4.	3						
5.	4						
6.	5						

- Plot Torque vs. Speed curves on a graph paper (approximated straight line plots)
- Compute B from the slope of Torque-Speed curve and average K_b from the table [Fig. 6].
- Repeat above for $E_a= 8$, $E_a= 10$, $E_a= 12$ and record the average values of motor parameters B and K_b.

4.4 Step Response

The dynamics of the motor is studied with the help of its step response. The various steps of this experiment are given below.

- Set 'MOTOR' switch to 'OFF'. Set 'RESET' switch to 'RESET'. Set 'LOAD' switch to 0 position.
- Connect E_a to the voltmeter and set it to 8V.
- Switch 'ON' the motor and measure E_g and the speed in rpm. These are the steady state generator voltage E_g and steady state motor speed N, respectively
- Set E_s to 63.2% of E_g measured above. This is the generator voltage at which the counter will stop counting.
- Switch 'OFF' the motor. Set 'RESET' switch to 'READY'.

- Now switch the motor ‘ON’. Record the counter reading as time constant in milliseconds.
- Repeat above with E_a = 10 V, E_a= 12 V and tabulate the results as shown below in Table 3.

Table - 3

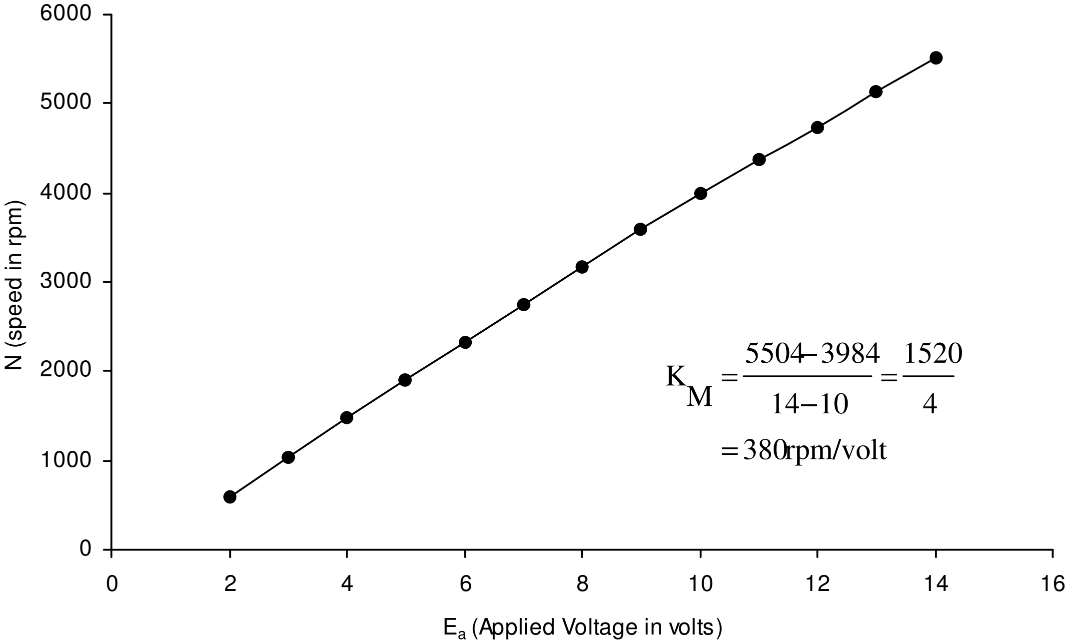
S.No.	E_a, volts	E_g, volts	N, rpm	E_s= 0.632.E_g volts	Time Constant τ_m msec	Gain Constant, $K_M = \frac{\pi N}{30E_a}$
1.						
2.						
3.						

- Substitute the values of K_M and τ_m in eqn. (6) and write down the motor transfer function.
- Using the average values of τ_m, B, K_b and R_a, calculate the motor inertia from eq. (7),

$$J = \tau_m \left(B + \frac{K_b^2}{R_a} \right)$$

4.5 Additional Experimentation

- It is also possible to get the data points for plotting the complete step response by setting $E_s = 0.1\ E_g,\ 0.2\ E_g, \ldots\ldots 0.9\ E_g$, and obtaining the time count to reach these values.
- Obtain τ_m at E_a = 10 volts with load switch set to 3. Note down the modified time constant and justify.
- The socket marked I_L on the motor unit may be used to measure the generator load current in terms of the voltage drop across a 5Ω resistance. This alongwith the open circuit generator voltage measurement, may be used to compute the actual electrical loading in watts and hence T_L. Further calculations may be made to establish the validity of the assumption, $K_b = K_T$, within reasonable experimental errors.



$$K_M = \frac{5504-3984}{14-10} = \frac{1520}{4}$$
$$= 380 \text{rpm/volt}$$

Fig 5(a) Motor Characteristics

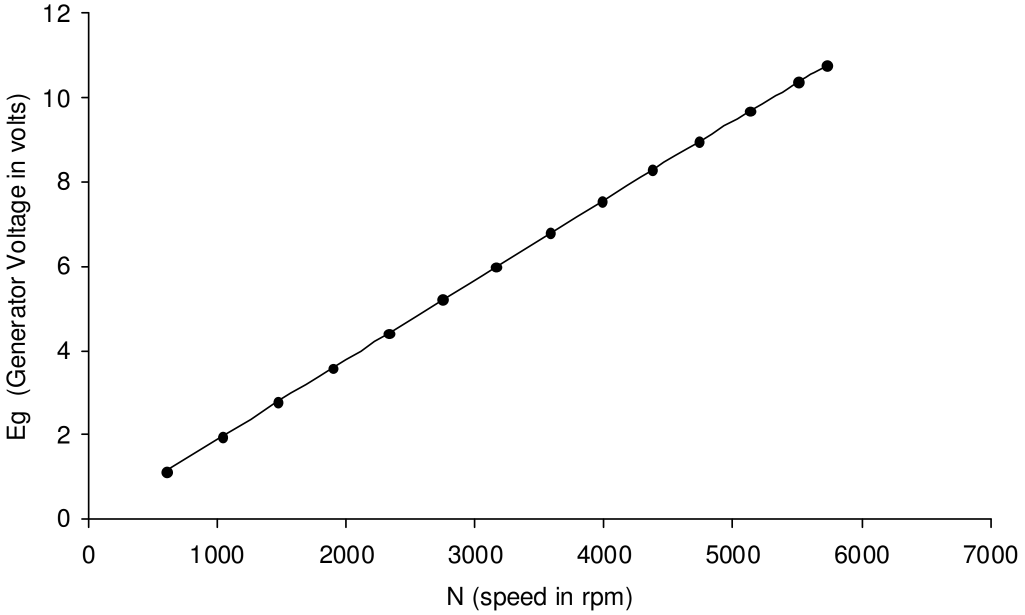


Fig 5(b) Generator Characteristics

5. RESULTS

Results obtained on a typical unit are given in Table 4 below for guidance only. Actual readings may vary from unit to unit.

(a) Armature Resistance, $R_a = 4.42\Omega$, Armature Inductance = 2.21 mH

(b) Motor and Generator Characteristics

S.No.	E_a, volts	I_a, mA	N, rpm	E_g , volts
1.	2	145	601	1.13
2.	3	143	1036	1.96
3.	4	138	1469	2.788
4.	5	137	1893	3.59
5.	6	135	2328	4.42
6.	7	136	2747	5.21
7.	8	139	3159	6.00
8.	9	141	3583	6.80
9.	10	143	3984	7.55
10.	11	146	4373	8.28
11.	12	150	4738	8.96
12.	13	153	5133	9.69
13.	14.	158	5504	10.37
14.	14.58	161	5728	10.77

From the plots of Fig 5(a) and 5(b), average values of motor and generator constants are:

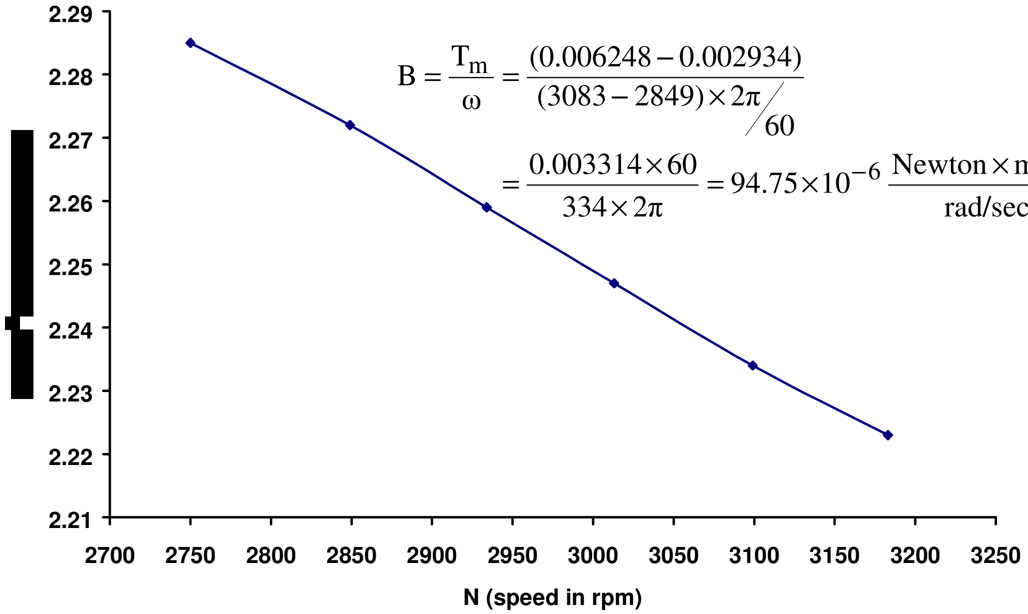
K_M = 380 rpm/volt

K_G = 0.00186 V/ rpm

(c) Torque-Speed Characteristics

$E_a = 8V$, $R_a = 4.42\Omega$

S.No.	Load Step	I_a, mA	N, rpm	$\omega = \frac{2\pi N}{60} = \frac{N\pi}{30}$ rad/sec.	$E_b = E_a - I_aR_a$ volts	$K_b = \frac{E_b}{\omega}$	$T_M = K_bI_a$ newton-m
1.	0	132	3183	333.3	7.41	0.02223	0.002934
2.	1	168	3099	324.5	7.25	0.02234	0.003753
3.	2	204	3013	315.5	7.09	0.02247	0.004583
4.	3	238	2934	307.2	6.94	0.02259	0.005376
5.	4	275	2849	298.3	6.78	0.02272	0.006248
6.	5	320	2750	287.9	6.58	0.002285	0.007312



$$B = \frac{T_m}{\omega} = \frac{(0.006248 - 0.002934)}{(3083 - 2849) \times 2\pi \big/ 60}$$

$$= \frac{0.003314 \times 60}{334 \times 2\pi} = 94.75 \times 10^{-6}\ \frac{\text{Newton} \times \text{meter}}{\text{rad/sec}}$$

Average $K_b = 22.53 \text{ x } 10^{-3} \ \frac{\text{volts}}{\text{rad/sec}}$

(From the slope of the curve) Fig. 6, coefficient of viscous friction,

$B = 94.75 \text{ x } 10^{-6} \ \frac{\text{newton} - \text{m}}{\text{rad/sec}}$

(d) Step response study E_a= 8V, E_g = 6.04 volts, and, τ_m = 77 msec

S.No.	E_a, volts	E_g, volts	N, rpm	E_s= 0.632.E_g volts	Time Constant τ_m msec	Gain Constant, $K_M = \frac{\pi N}{30E_a}$
1.	8	6.04V	3186	3.817V	77 msec	41.70
2.	10					
3.	12					

E_s = 0.632 x 6.04 = 3.817 V and $K_M = \frac{\pi}{30}.\frac{N}{E_a} = \frac{\pi}{30}.\frac{3186}{8} = 41.70$ rad/sec.

$$\text{Thus,} \quad J = \tau_m \left(B + \frac{K_b^2}{R_a} \right)$$

$$= 0.077 \left[94.75 \times 10^{-6} + \frac{(22.53 \text{ x } 10^{-3})^2}{4.42} \right]$$

$$= 16.14 \text{ x } 10^{-6} \ \frac{\text{N.m}}{\text{rad/sec}^2}$$

and motor transfer function is

$$G(s) = \frac{\omega(s)}{E\ (s)} = \frac{K_M}{s\tau_m + 1} = \frac{41.70}{0.077s + 1} = \frac{541.55}{s + 12.98}$$

6. REFERENCES

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Laboratory Manual

for

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D.C. POSITION CONTROL DCP - 01

1. OBJECT

To study the performance characteristics of a d.c. motor angular position control system.

2. EQUIPMENT DESCRIPTION

A major portion of any first course on automatic control system invariably revolves around the study of d.c. position control systems. Experimental work in this area has however been confined to analog simulated systems, e.g. through our 'Linear System Simulator' or similar other units. The biggest advantage of this approach is the unlimited flexibility and near perfect operation of the simulated systems leading to a close correlation between theoretical and experimental results, however, the student is denied the feel of a physical electromechanical system. The present unit has been designed with this objective in mind. Despite the constraints like friction, dead zone, nonlinearities due to amplifier saturation and motor current limiting, and low speed of response associated with any mechanical system, the student has been provided with enough opportunity for experimentation on a working system. The panel diagram in Fig. 1 shows the various built-in subsystems which are now described.

2.1 Signal Sources

- Angle command (continuous): obtained through a potentiometer with a calibrated disk attached.
- Angle command (step): available through a toggle switch. Automatic synchronisation with waveform capture circuit is provided.

2.2 Motor Unit

The position control is achieved through a good quality permanent magnet d.c. gear motor. The specifications of the motor are :

- Operating voltage : 24Vdc
- No load current : 0.16A
- Full load current : 2.8A
- Rated speed : 40 rpm
- Torque (basic) : 1.48 Kg-cm

However in DCP-01 we are using this motor at only 12Vdc.

Angular position of the motor shaft is sensed by a special 360° rotation potentiometer attached to it. A calibrated disk mounted on the potentiometer indicates its angular position in degrees. In addition to this, a small tachogenerator attached to the motor shaft produces a voltage proportional to its speed which is used for feedback.

All the above components, viz. the motor, potentiometer, tachogenerator etc. are fitted inside the 'motor unit'. Transparent panels provide a good view of the interior. The motor unit is connected to the rest of the system through a 9-pin D-type connector and cables.

2.3 Main Unit

The main unit houses the command circuit, the error detector, the gain controls of the forward path and tachogenerator channels, the power stage and the waveform capture/display unit. Different experiments are performed by appropriate settings of the controls as explained later. Description of the above blocks is given next.

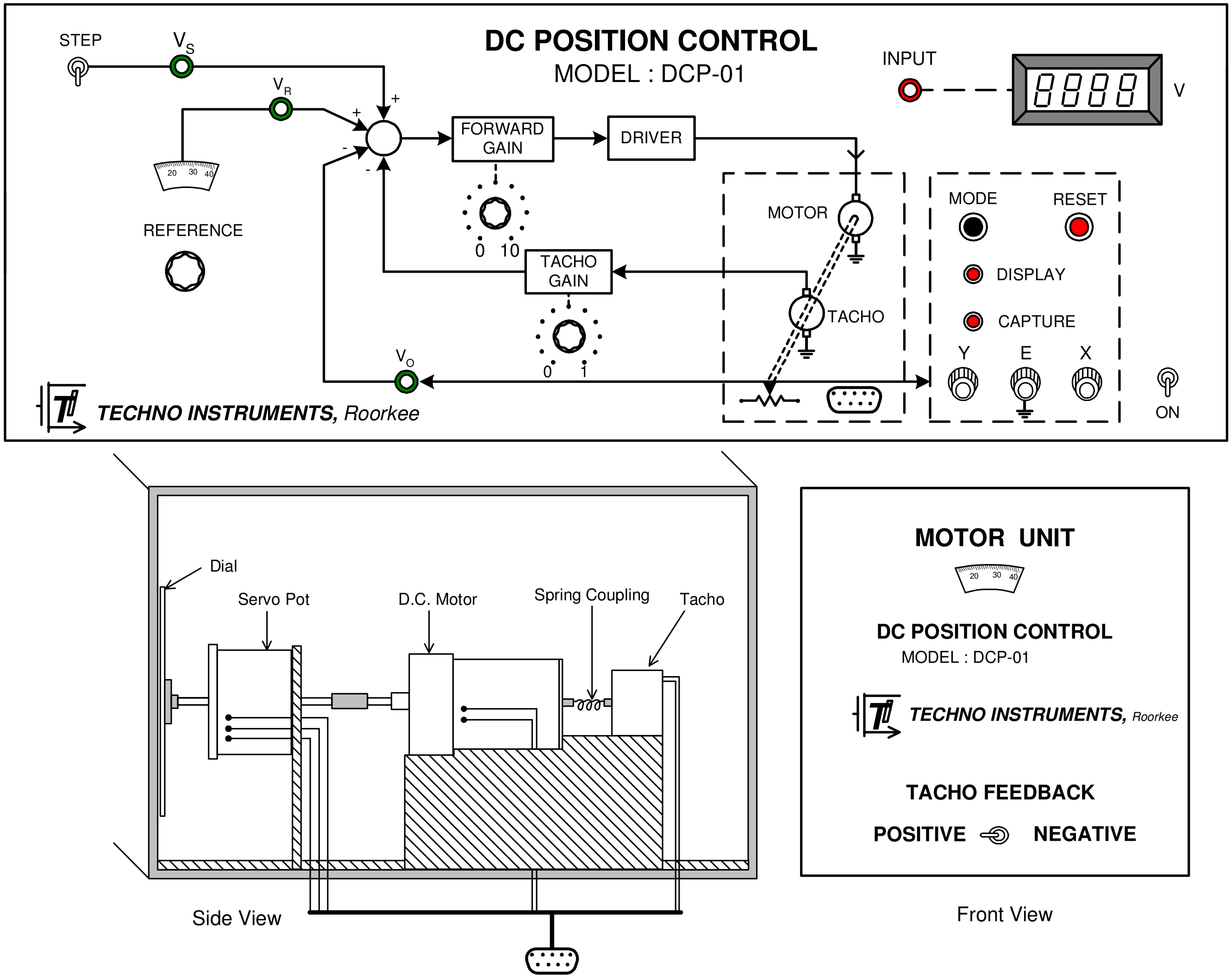


Fig.1 :Panel diagram of D.C. Position Control, Model DCP-01

(a) Command: Two operating modes have been provided in the system. When a continuous command is given by the rotation of a potentiometer through a certain angle, the closed loop system responds by an identical rotation of the motor shaft. Alternatively, a step command equivalent to about 150 degrees may be given by a switch. This is used for quantitative studies of the step response.

(b) Error detector: This is a 4-input 1-output block. Two of the inputs are meant for command signals and the remaining two inputs, having 180° phase shift, are used for position and velocity feedback signals.

(c) Gain blocks: The forward path gain is adjustable from 0 to 10 and the tachogenerator channel gain may be varied from 0 to 1. The gains may be read from the markings on the panel.

(d) Driver: The driver is a unity gain complementary symmetry power amplifier suitable for running the motor upto full power in either direction. A current limiting circuit ensures safety of the power transistors during motor starting and direction reversal.

(e) Waveform Capture/Display unit: The time response of a mechanical system like the present one is usually too slow for a CRO display, except on a storage oscilloscope. Alternatively an X-Y recorder could be used to get a hard copy which may subsequently be studied quantitatively. Both these options are quite expensive for a usual undergraduate laboratory. The waveform capture/display unit is a microprocessor based card which can 'capture' the motor response and then 'display' the same on any ordinary X-Y oscilloscope for a detailed study. The stored waveform is erased whenever another waveform is captured, or the unit is reset.

2.4 Power Supply

The set-up has a number of IC regulated supplies which are permanently connected to all the circuits. No external d.c. supply should be connected to the unit.

Capabilities of this unit include an evaluation of the performance of the position control system for different values of forward gains. Also the effect of tachogenerator feedback on system stability forms an important study. Effect of non-linearity, so common in all practical systems, may be readily observed by the student. In all the cases the response is stored and can then be displayed on an ordinary measuring oscilloscope.

3. BACKGROUND SUMMARY

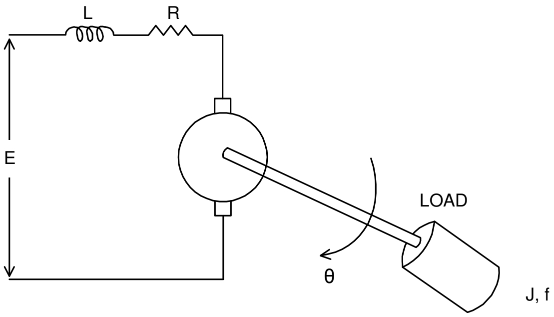
Second order systems are studies in great detail in any course on linear control system. The reason for this is that a large number of higher order practical control systems may be approximated as a second order system while neglecting less dominant modes, nonlinearities like dead zone, saturation, hysteresis etc., assuming these to have little effect on the performance. Also second order systems lend themselves to a simple and accurate mathematical analysis. In the following description we shall follow the above strategy. At the end however, the imperfections due to nonlinearities shall be pointed out.

3.1 Position Control - a second order system

A second order system is represented in the standard form as,

$$G(s) = \frac{\omega_n{}^2}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

where ζ is called the damping ratio and ω_n the undamped natural frequency. Depending upon the value of ζ, the poles of the system may be real, repeated or complex conjugate which is reflected in the nature of its step response. Results obtained for various cases are :



$$\frac{\theta(s)}{E(s)} \approx \frac{Km}{s(sT+1)}$$

Fig. 2 : Transfer function of D.C. Motor

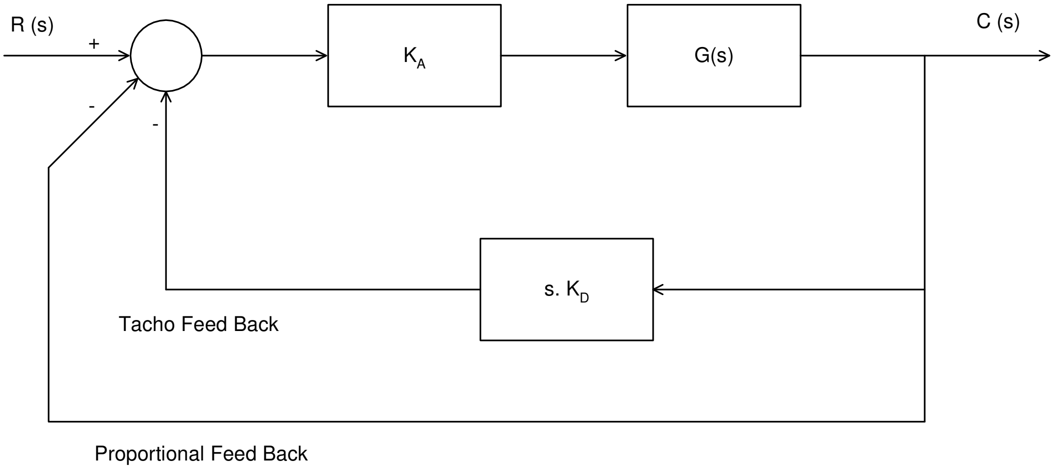


FIG. 3 : Simplified block diagram

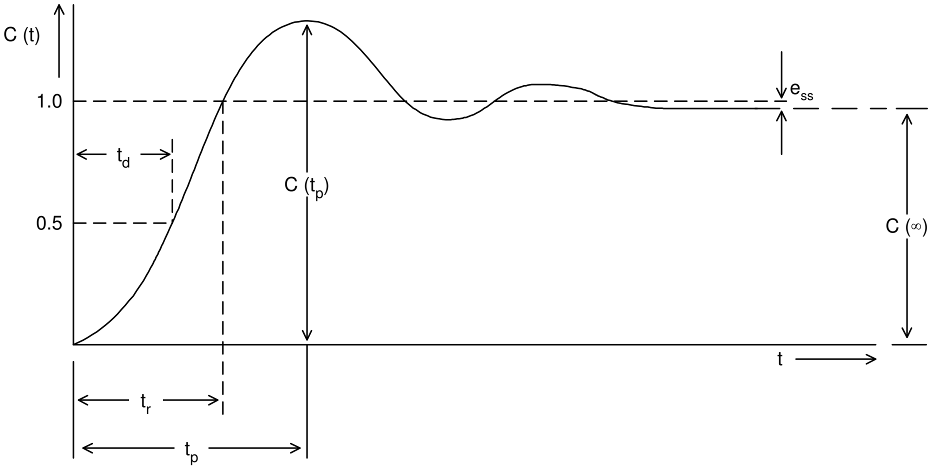


Fig.4 : Normalized step response of the second order system

(a) underdamped case $(0<\zeta<1)$

$$c(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{(1-\zeta^2)}}\sin(\omega_d t+\tan^{-1}\frac{\sqrt{(1-\zeta^2)}}{\zeta}) \qquad (1)$$

where, $\omega_d=\omega_n\sqrt{(1-\zeta^2)}$ is termed the damped natural frequency. A sketch of the unit step response for various values of ζ is available in the text books.

(b) Critically damped case $(\zeta=1)$

$$c(t) = 1-e^{-\omega_n t}(1+\omega_n t) \qquad (2)$$

(c) Overdamped case $(\zeta>1)$

$$c(t)=1-\frac{\omega_n}{2\sqrt{(1-\zeta^2)}}\left(\frac{e^{-s_1 t}}{s_1}-\frac{e^{-s_2 t}}{s_2}\right) \qquad (3)$$

where $s_1=(\zeta+\sqrt{(\zeta^2-1)})\omega_n$ and $s_2=(\zeta-\sqrt{(\zeta^2-1)})\omega_n$

Referring to Fig. 2, the transfer function G(s) of an armature controlled d.c. motor may be derived as [1],

$$\frac{\theta(s)}{E(s)} = G(s) = \frac{K_m}{s(sT+1)}$$

where K_M is Motor gain constant, and T the Mechanical time constant.

Considering proportional feedback only, the close loop transfer function of the system of Fig. 3 may be obtained as,

$$\frac{C(s)}{R(s)} = \frac{K_A G(s)}{1+K_A G(s)} = \frac{K_A K_m / T}{(s^2+s/T+K_A K_m / T)} \qquad (4)$$

This gives unit step response similar to equations (1), (2) or (3) depending upon the value of K_A. Thus the response of the position control system can be altered by varying the amplifier gain K_A, and a 'satisfactory' performance may usually be obtained. This leads to the concept of performance characteristics as defined on the step response of an underdamped second order system in Fig. 4 and explained in brief here.

(i) ***Delay time***, t_d, is defined as the time needed for the response to reach 50% of the final value.

(ii) ***Rise time***, t_r, is the time taken for the response to reach 100% of the final value for the first time. This is given by

$$t_r=\frac{\pi-\beta}{\omega_d}, \text{ where } \beta=\tan^{-1}\frac{\sqrt{(1-\zeta^2)}}{\zeta}$$

(iii) ***Peak time***, t_p, is the time taken for the response to reach the first peak of the overshoot and is given by

$$t_p=\frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$$

(iv) ***Maximum overshoot***, M_p, is defined by

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

(v) ***Settling time***, t_s, is the time required by the system response to reach and stay within a prescribed tolerance band which is usually taken as ±2% or ±5%. An approximate calculation based on the envelops of the response for a low damping ratio system yields

t_s (±5% tolerance band) = $3/\zeta\omega_n$
t_s (±2% tolerance band) = $4/\zeta\omega_n$

Another important characteristic of a closed loop system is the steady state error, e_{ss}. For unity feedback systems e_{ss} is defined as

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} \{r(t) - c(t)\}$$

A simpler way to calculate steady state error without actually computing the time response is available in the complex frequency domain. Application of the final value theorem of Laplace Transform to unity feedback system gives,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

Steady state error may be obtained for various inputs (step, ramp, parabolic) and systems of various type numbers (number of poles at origin). A summary of the results of the above calculations may be seen in [11]. To facilitate the calculations, error coefficients are defined as

Position error coefficient, $K_p = \lim_{s\to 0} G(s)$
Velocity error coefficient, $K_v = \lim_{s\to 0} sG(s)$
Acceleration error coefficient, $K_a = \lim_{s\to 0} s^2 G(s)$

- The position control system has a second order transfer function in the standard form.
- The system should not have any steady state error for step input.
- The transient response of the system is affected by the value of K_A. A higher value of K_A should result in larger overshoot.

3.2 Tachogenerator feedback

It may be intuitively obvious that availability of a single adjustable parameter K_A in the position control system is likely to meet only one of the performance characteristics. In most cases however one is interested in at least two specifications simultaneously e.g. steady state error and the damping factor or peak overshoot. In an electromechanical system this is conveniently achieved through a tachogenerator feedback.

Considering the tachogenerator feedback path also active in Fig. 3, the closed loop transfer function is obtained as

$$\frac{C(s)}{R(s)} = \frac{K_A G(s)}{1 + K_A G(s)(1 + K_D s)} = \frac{K_A K_m / T}{s^2 + (1 + K_A K_m K_D)(s/T) + (K_A K_m)/T}$$

It is easily seen that the steady state error to unit ramp is given by

$e_{ss} = 1/K_A K_m$, and the damping ratio by

$\zeta = (1+K_A K_m K_D)/2\sqrt{}(TK_A K_D)$

Thus the specification of e_{ss} and ζ may be met simultaneously by a proper choice of K_A and K_D.

3.3 System Imperfections

All practical systems are imperfect to some extent. As a result of this, the actual system response differs from the ideal response of Fig. 4, which is valid for a second order linear system. Some of the contributing factors relevant to the present set-up are :

(a) ***Saturation of armature current*** - necessary to protect the driver from high currents when the motor starts or reverses its direction. This implies limiting the maximum control effort for large errors leading to a slower response.

(b) ***Amplifier saturation*** - has effects similar to above although the saturation is now a circuit limitation.

(c) ***Dead zone*** - caused by a minimum voltage below which the motor would not start due to the friction of the brushes and bearings. As a result of this the steady state error may be larger than expected.

(d) ***Nonlinear tachogenerator and motor characteristics*** - due to manufacturing inaccuracies.

(e) ***System order*** - may be actually more than two, due to load characteristics, delays and filters used.

An accurate analysis taking into account the above mentioned imperfections would certainly prove to be exceedingly complex. The experiments which follow therefore consider the system as it is, study the response and the effect of tachogenerator feedback on the response. A qualitative comparison of the result of experiment with the theoretical predictions for a second order linear system should be of great interest.

4. EXPERIMENTAL WORK

The experiments suggested below enable the reader to study the performance of the closed loop system with proportional feedback and closed loop system with combined proportional and techogenerator feedback. Idea of dead zone and its effect on steady state error is also introduced. A special provision has been made in the set-up to store and display a response of the system - a need which occurs quite frequently. The operation of this waveform capture/display provision is described first.

4.1 Waveform Capture/Display

This card is designed to automatically store the response of the system in a RAM whenever a step input is given. The stored response is then displayed on the CRO. Steps for its operation are as given below:

(a) Power ON the system and/or press the RESET switch - unit goes into DISPLAY the axes and shows the RAM contents (zero at present).

(b) Press the MODE switch - the unit becomes ready to capture the step response.

(c) Applying step input now starts the storage. At the end of the capture cycle, the mode automatically shifts to DISPLAY and the response waveform is seen on the CRO.

(d) Storage of a new response or pressing the RESET switch erases the current waveform.

(e) The time scale of the display may be calibrated by feeding the X-output (sawtooth) of the unit to the Y-input of the CRO and determining its time period and amplitude.

4.2 Closed loop study (Also see the Note at end, on page 10)

(a) Position control through CONTINUOUS command

- Ensure that the step command switch is OFF
- Starting from one end, move the COMMAND potentiometer in small steps and observe the rotation of the response potentiometer.
- Record and plot θ_R, V_R, θ_0 and V_0 for a few values of K_A.
- Calculate $\Delta\theta_R$ and $\Delta\theta_0$ (taking initial readings as nominal values) and plot. Also calculate the errors ($\Delta\theta_R$-$\Delta\theta_0$), (ΔV_R-ΔV_0) at each step. Justify the presence of errors and their variation with K_A.

(b) Position control through STEP command

- *Ensure that the tachogenerator feedback switch on the MOTOR UNIT is set to NEGATIVE.*
- Adjust the reference potentiometer to get V_R=0.
- Set K_A to 2.
- Connect the CRO, calibrate the time scale, sec. 4.1(e), and switch to CAPTURE mode.
- Apply STEP input. Wait till storage is complete and the response is displayed. Trace the waveform from CRO.
- Compute M_p, ζ, t_p, t_r and the steady state error.
- Repeat for K_A = 3,4,....
- Now set K_A=6, and choose various values of K_D=0.1, 0.2... and repeat the above observations.
- Tabulate the results as shown in the next section and discuss :
    - ◊ variation of maximum overshoot, rise time and steady state error with forward gain.
    - ◊ effect of tachogenerator feedback on maximum overshoot, rise time and stability.
    - ◊ effect of dead zone and saturation on step response.
- Compare your results with theoretical predictions assuming a second order system.
- A set of observations with POSITIVE tachogenerator feedback may also be taken in the same manner as above.

5. TYPICAL RESULTS

Typical results obtained on a similar unit are next given for guidance. The reading and result have all been obtained using the waveform capture and other built-in facilities of the unit. However, a set of step response recording obtained through TEKTRONIX Storage CRO Type TDS-210 and its PC interface are shown on the facing pages of 8 & 9, purely for general information

(a) Manual operation of the position control

$K_D = 0$, Tachogenerator channel disabled

$K_A = 5$

S. No.	θ_R deg	$\Delta\theta_R$ deg	θ_0 deg	$\Delta\theta_0$ deg	$\Delta\theta_R$-$\Delta\theta_0$ deg	V_R volt	V_0 volt	ΔV_R-ΔV_0 volt
1.	0	-	5	-	-	0	0	0
2.	30	30	31	26	4	0.20	0.13	0.07
3.	60	60	62	57	3	0.87	0.90	-0.03
4.	90	90	95	90	0	1.43	1.48	-0.05
.								
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The measured values of VR have negative signs which have not been inverted in the internal circuitry for technical reasons. These may however be read as positive and calculation should be made with positive values

(b) Calibration of X-output

In the DISPLAY mode with X-output connected to the Y-input of CRO, a sawtooth waveform is seen. On measurement,

Amplitude of sawtooth = 5.6 volts.

Time duration of the main linear part = 39 msec.

X-output scale factor is thus 6.96 msec/volt

The X-output waveform above consists of axis display part and waveform display part. The latter is identified by a much longer time duration which has been measured above.

(c) Step response of the position control without tachogenerator feedback

Set K_D=0

V_s=2.5 V (internally set)

S. No.	K_A	M_p %	t_p msec	t_r msec	ζ	e_{ss} volt	ω_n rad/sec
1.	5	16.8	10.5	4.2	0.493	0.12	343.9
2.							
3.	7	20.8	6.96	3.48	0.447	0.0	504.6
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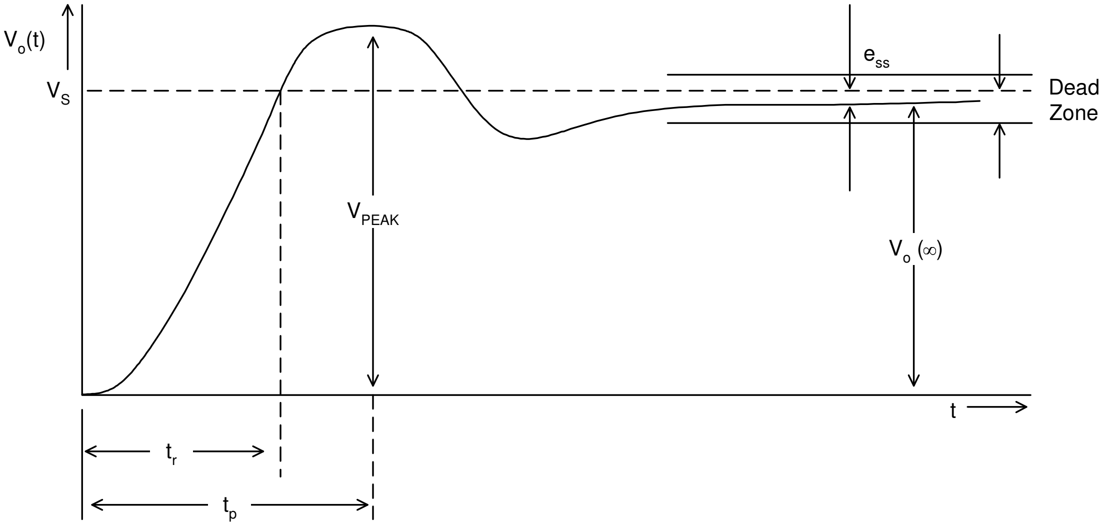

Fig. 5 : Typical step response of the position control system

Referring to Fig. 5,

$e_{ss}=V_s-V_0$, where V_s and V_0 may be measured by DVM

$$M_P = \frac{V_{PEAK} - V_0(\infty)}{V_0(\infty)} \times 100\%$$

t_p, t_r may be obtained from CRO

ζ may be calculated from M_p using the standard relation

$M_p = \exp\left(-\pi\zeta/\sqrt{(1-\zeta^2)}\right)$

ω_n is calculated from the expression of t_p $\{=\pi/\omega_n\sqrt{(1-\zeta^2)}\}$

The closed loop and open loop transfer functions of the system may now be written as,

Closed loop : $\dfrac{\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2} = \dfrac{2.54\times10^5}{s^2+4.51\times10^2 s+2.54\times10^5}$

open loop (excluding K_A) : $\dfrac{1}{K_A}\dfrac{\omega_n{}^2}{s(s+2\zeta\omega_n s)} = \dfrac{2.54\times10^5}{7\times s(s+4.51\times10^2 s)}$

- The open loop transfer function (excluding K_A) comes out to be different for different readings - the system is not actually a second order function.
- The peaks of the response curves are flattened - the motor has dead zone.
- The peak overshoot does not increase significantly with K_A - motor armature currents is restricted.

(d) *Step response of the position control with tachogenerator feedback*

$K_A=7$

$V_s=2.5$ volts (internally set)

S. No.	K_P	M_p %	t_p msec	t_r msec	ζ	e_{ss} volt	ω_n rad/sec
1.	0	20.80	6.96	3.48	0.447	0.1	504.6
2.	0.1	12.44	8.10	4.17	0.552	0.1	465.1
3.	0..2	0	-	5.22	1	0	-
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M_P, t_p, ζ and e_{ss} may be obtained as outlined in (c) above.

- The tachogenerator feedback is seen to reduce M_P and increase ζ. Relative stability is improved.
- There is an increase in t_r and t_p. The system becomes slower.
- The steady state error remains unchanged.

NOTE: Under certain operating conditions, the motor may start continuous uncontrolled rotation. This is due to a very small gap (approx. 5°) in the response potentiometer which is easily overshot by the motor, due to its inertia. In such a situation normal operation may be restored by decreasing the gain or by changing the position of the command potentiometer.

6. REFERENCES

[1] Control System Engineering - I. J. Nagrath and M. Gopal, Wiley Eastern Ltd.

[2] Modern Control Engineering - K. Ogata, Prentice Hall of India Pvt. Ltd.

[3] Automatic Control System - B.C. Kuo, Prentice Hall of India Pvt. Ltd.

Laboratory Manual

for

Control System Lab

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STUDY OF MAGNETIC LEVITATION SYSTEM ML-01

1. OBJECTIVE

Theoretical and experimental study of a magnetic levitation setup, an inherently unstable system.

2. BACKGROUND SUMMARY

The magnetic levitation system consists of an electromagnet which pulls an object (a magnetic material) in an upward direction, in the presence of downward gravitational force on it (Fig.1). If the magnet current i is adjusted to satisfy the condition, f = mg, the object should, at least theoretically, remain suspended in air. In a practical situation, however, even the smallest disturbance would dislocate the balance and the object would either stick to the magnet or fall down to ground. Logically therefore the current i needs to be continuously adjusted to keep the object freely suspended in air. This task is impossible to be achieved manually , and therefore needs a feedback control loop.

The basic feedback control scheme is shown in Fig.2. The idea here is to monitor the position of the object continuously and adjust magnet current automatically to ensure the upward force, f, exactly balances the weight of the object, mg, at all times. It will however be seen later that due to the unstable dynamics of the object, the automatic control scheme of Fig.2 is not workable and hence a more elaborate controller is required. A linearized model of the system is developed next.

2.1 System Model

The electromagnet pulls the object (iron ball) with a force,

$$f(x,i) = c\left(\frac{i}{x}\right)^2 \text{, and } c = \frac{L_0 x_0}{2} \qquad \text{....(1)}$$

Here,

x is the distance between the magnet and the object

i is the current in the magnet coil

c is the constant of proportionality

L_o is the additional inductance of the magnet coil due to the object placed at the nominal position, $x = x_0$.

The equation of motion of the object is given by

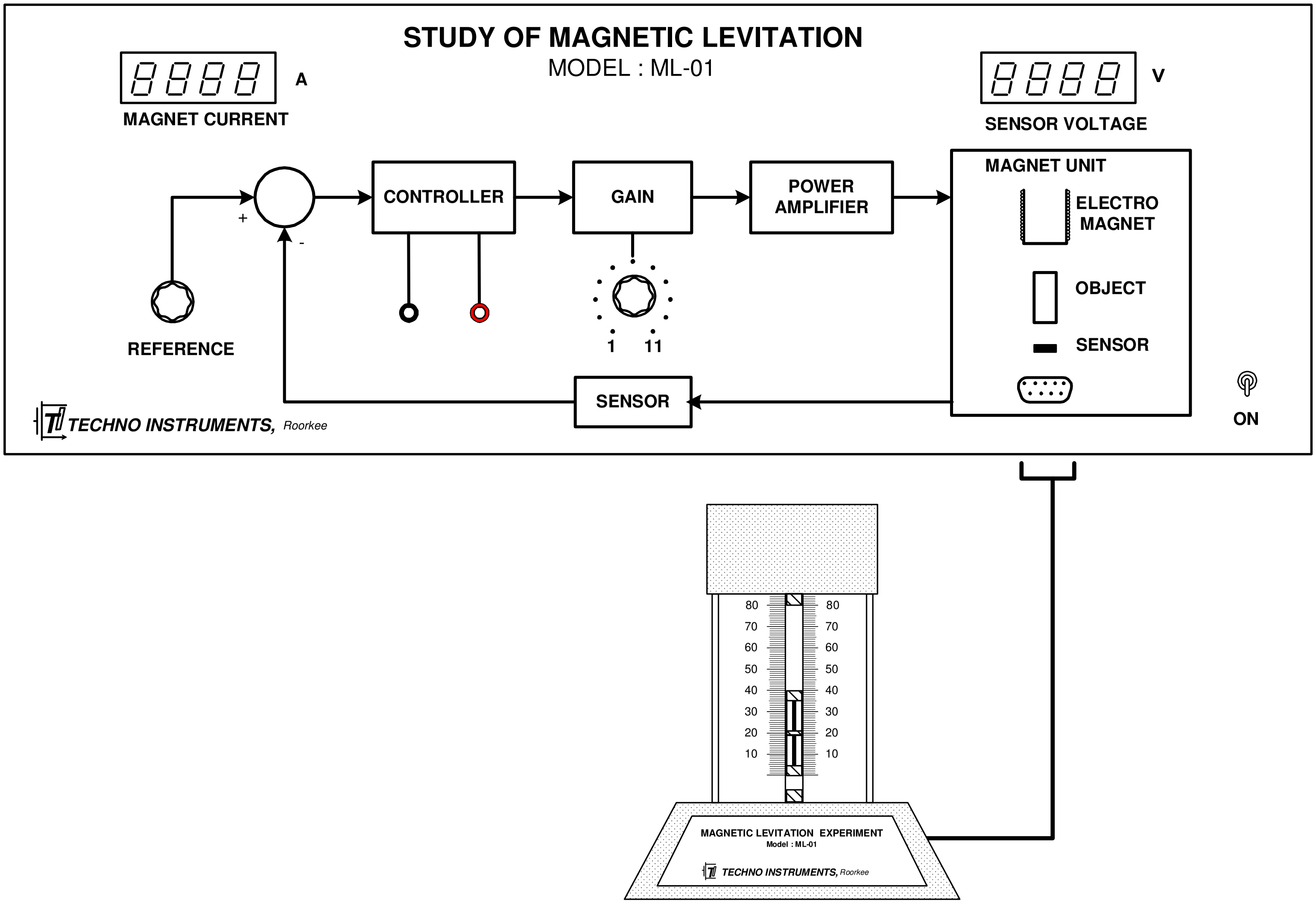
$$m\ddot{x} = -c\left(\frac{i}{x}\right)^2 + mg\text{, where} \qquad \text{....(2)}$$

m is the mass of the object, and

g is the acceleration due to gravity.

A linearized form of the above equation may be derived as,

$$m\ddot{\tilde{x}}(t) = -c\left(\frac{i_0}{x_0}\right)^2\left\{1+2\left(\frac{\tilde{i}(t)}{i_0}-\frac{\tilde{x}(t)}{x_0}\right)\right\} + mg \qquad \text{....(3)}$$



Panel Drawing Magnetic Levitation, Model ML-01

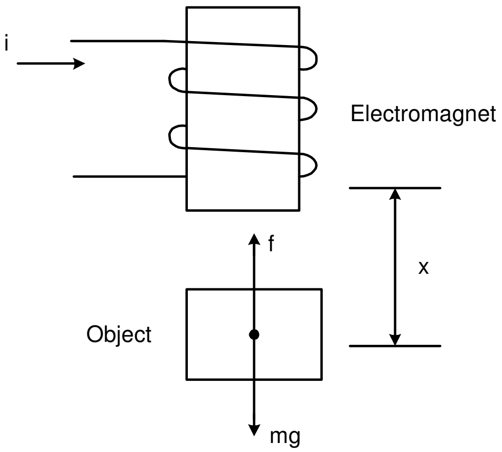


Fig.1 Basic system

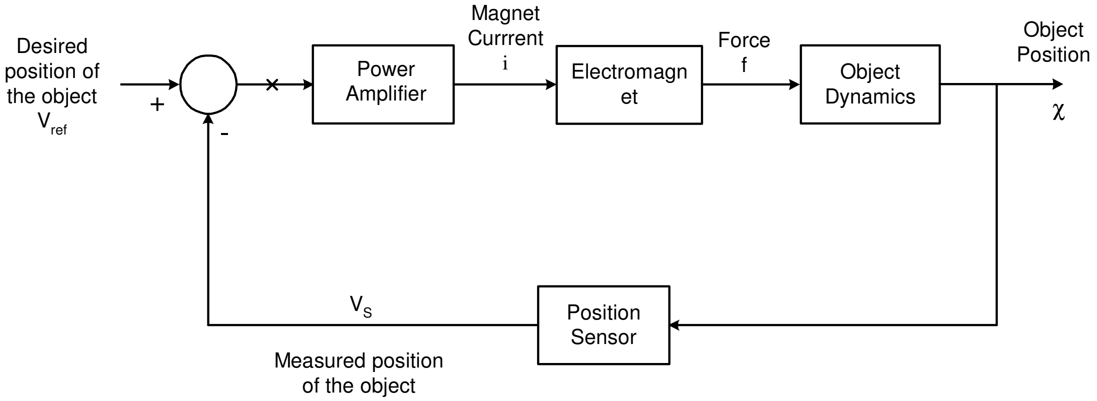


Fig.2 Automatic Scheme for Control

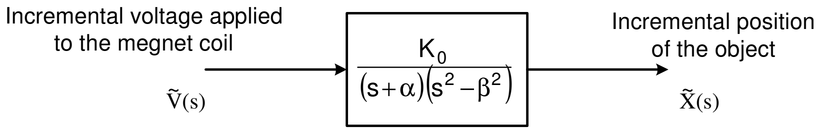


Fig.3 Open Loop System Model

where, $\tilde{x}(t)$ and $\tilde{i}(t)$ are the incremental displacement and incremental magnet current around their nominal values, x_0 and i_0. Since the object is assumed to be at rest at $x = x_0$ with $i = i_0$,

$$c\left(\frac{i_0}{x_0}\right)^2 = mg \text{ , which leads to}$$

$$m\ddot{\tilde{x}}(t) = -\frac{2i_0 c}{x_0^{\;2}} \cdot \tilde{i}(t) + \frac{2i_0^{\;2}c}{x_0^{\;3}}\tilde{x}(t)$$

Taking Laplace transform and neglecting initial condition,

$$\frac{\tilde{X}(s)}{\tilde{I}(s)} = \frac{\dfrac{-2ci_0}{mx_0^{\;2}}}{s^2 - \dfrac{2ci_0^{\;2}}{mx_0^{\;3}}} \quad , \qquad \text{….(4)}$$

is obtained as the dynamics of the object.

The current – voltage relation of the magnet is given by

$\tilde{v}(t) = R\,\tilde{i}(t) + L\,\dfrac{d\,\tilde{i}(t)}{dt}$, where $\tilde{v}(t)$ and $\tilde{i}(t)$ are the incremental values of voltage applied to and the current flowing into the electromagnet, and R, L are the coil parameters. Taking Laplace transform and neglecting initial conditions,

$\tilde{V}(s) = (R + sL)\,\tilde{I}(s)$, and

combining the above equations, the system dynamics is given by the transfer function

$$\frac{\tilde{X}(s)}{\tilde{V}(s)} = G(s) = \frac{-\dfrac{2ci_0}{mLx_0^{\;2}}}{\left(s+\dfrac{R}{L}\right)\left(s^2 - \dfrac{2ci_0^{\;2}}{mx_0^{\;3}}\right)} = -\frac{K_0}{(s+\alpha)(s^2-\beta^2)} \qquad \text{….(5)}$$

2.2 Controller Scheme

The above equation may be represented by the block diagram of Fig. 3 which is the open loop system.

It is clear that the open loop system above is unstable, due to the pole at $s = \beta$, in the right half of the s-plane. Also, connecting a feedback loop on the line of Fig.2 will not lead to a stable closed loop system for any value of forward path gain as may be seen from the root locus diagram of Fig.4. This obviously is a proportional controller, which will not stabilize the system. In case however a proportional – derivative (PD) controller is used, the revised root locus diagram of Fig.5 indicates the possibility of stabilizing the system and also achieving good transient performance for some range of values of forward path gain.

The PD controller is assumed to have the transfer function, $G_c = (1 + T_d s)$

The present experiment involves determination of the system parameters, K_0, α and β experimentally, and then to design T_d, the parameter of the controller and choose an appropriate gain.

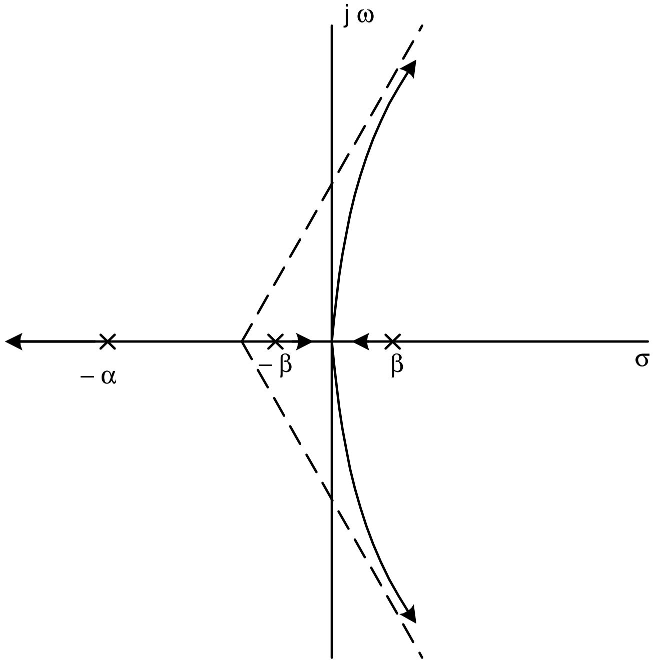


Fig.4 Root Locus with Proportional Controller

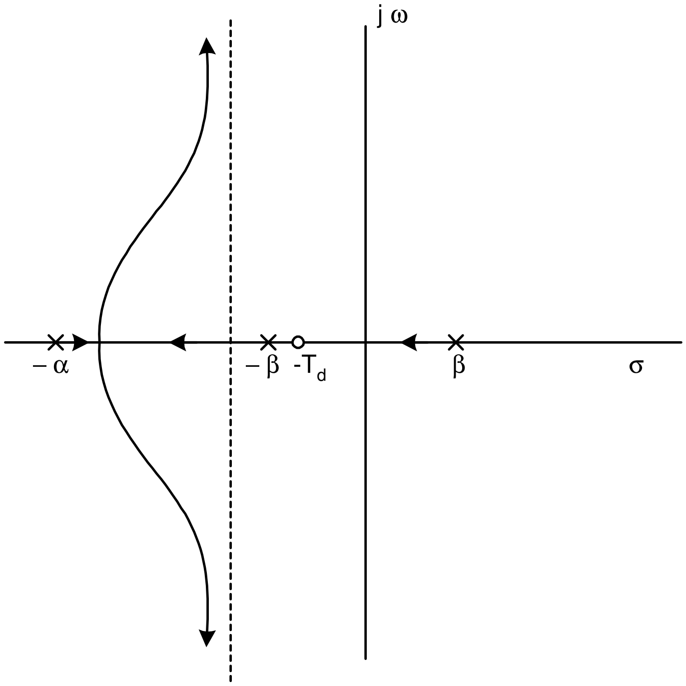


Fig.5 Root Locus with PD Controller

2.3 Parameter determination

(a) The value of α can be calculated from the magnet parameters, R and L, and using the expression $\alpha = R/L$.

(b) Since the open loop system, eqn.(5) is unstable, no experimentation is possible directly on it. An alternative method is to use an adhoc setting of the PD controller to result in stability. Then the force balance equation, $c\left(\frac{i}{x}\right)^2 = mg$,is used to calculate an average value of c from measurements of i and x around the nominal equilibrium point i_0, x_0. The mass of the object, m, is known beforehand.

The constant K_0 and β are then computed from

$$K_0 = \frac{2ci_0}{mLx_0^2} \text{ and } \beta = \sqrt{\left(\frac{2ci_0^2}{mx_0^3}\right)}$$

3. SYSTEM DESCRIPTION

The hardware involved in the various blocks of Fig.2 are described in this section.

3.1 Suspended Object

This comprises of two disc magnets M_1 and M_2 fixed on the two ends of a height plastic cylinder (Fig. 6). While M_1 is used for providing upward force to pull the object, M_2 is used for generating the necessary position information through the hall sensor fixed on the base. A cylinder made of plastic material ensures proper view and protected movement of the object. Note that our earlier analysis refers to iron as the suspended object, in the actual model a magnet M_1 is used. This ensures a better control when the object goes close to the electromagnet. Also since the thickness of a magnet is much smaller that x_0, the calculation are not too much in error. Finally, since the system is highly non-linear and we are linearizing around the operating point x_0, i_0, the whole analysis is reasonably accurate even for a magnet object.

3.2 Electromagnet

A powerful electromagnet is mounted on the top of the guide rod. It exerts variable attraction force on M_1 depending on the current supplied to it. The second magnet M_2, being farther away, is assumed to have a negligible effect on the force. The electromagnet is characterised by its winding resistance R and inductance L.

3.3 Power Amplifier

It is a complementary symmetry power amplifier having an internal gain of 2, and capable of providing up to 3 amp. to the electromagnet. The external gain of the circuit may be varied form 1 to 11 using the potentiometer on the panel.

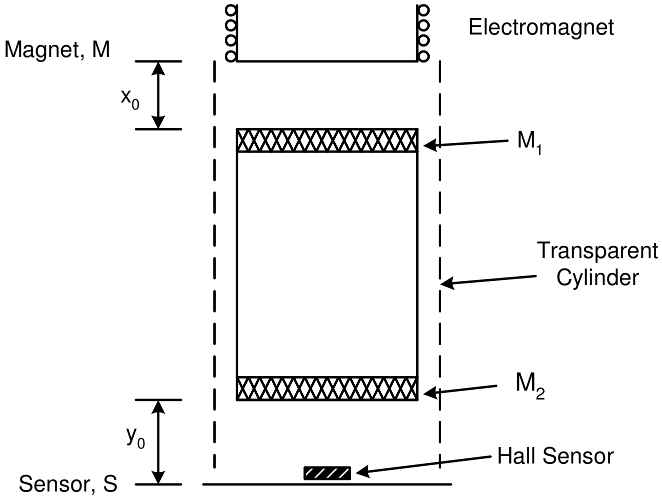


Fig.6 Mechanical Arrangement of the Object

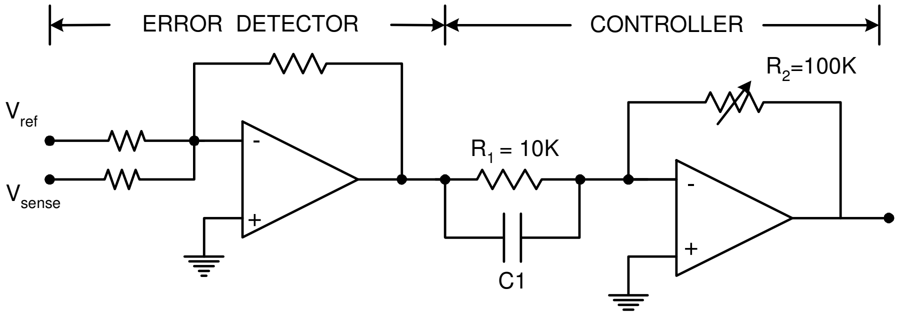


Fig.7 Electronic Circuit of the System

3.4 Position Sensor

It is a hall effect device, which generates an electrical signal proportional to its distance from M_2. The sensor output is thus a measure of the position of the suspended object. Note that due to the highly non-linear nature of the overall system, we are basically concerned with small deviations around the equilibrium point. This enables us to represent the sensor as a constant gain for computation purposes.

3.5 Error detector and controller

The circuit diagram of this section is shown in Fig. 7 . It performs the tasks of,

(i) Placing the object to a suitable position which may be taken as the 'nominal position'. This is done by adjusting the reference signal, V_{ref}.

(ii) Providing a PD controller to ensure system stability, and a simple calculation shown the controller transfer function to be, $\frac{R_2}{R_1} \cdot (1 + s R_1 C_1)$.

(iii)Setting the forward path gain to an appropriate value.

4. EXPERIMENTS AND RESULTS

The experiments conducted on a typical unit and the results are described below in some detailed. The student is expected to repeat the steps on the unit available to him and go through the design. Actual results are likely to be somewhat different in each case.

4.1 System Identification

Standard Parameter values of a typical unit are:

Magnet Inductance, $\mathbf{L = 8.69 \times 10^{-3}}$ **h**

Magnet Resistance, $\mathbf{R = 4.95\ \Omega}$

Mass of the Object, $\mathbf{m = 10 \times 10^{-3}}$ **Kg**

(a) Based on the parameters of the magnet, the time constant of the coil may be calculated as,

$$\alpha = \frac{4.95}{8.69 \times 10^{-3}} = 569.62 \text{ sec}^{-1}$$

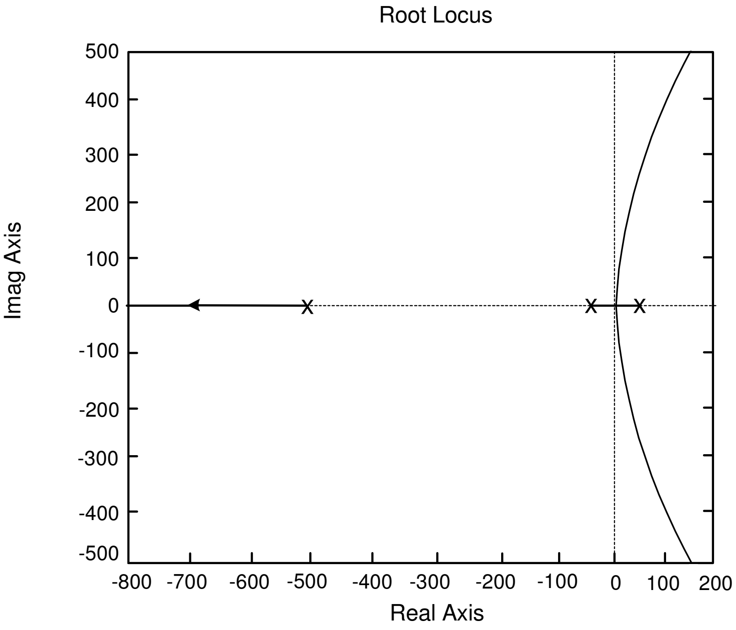
(b) The steps below are now followed to determine the remaining parameters.

Step.1 Connect the PD controller (C_1 = 2μF) and close the feedback loop. Set forward gain to a medium value say 4. The object should now be suspended freely in air by adjusting V_{ref}.

Step.2 Adjust the reference input to move the object in the gap between the magnet and the sensor. This is the nominal position of the object [x_0 is the distance between the object (top marker) and the electromagnet, M and i_0 is the electromagnet current.]

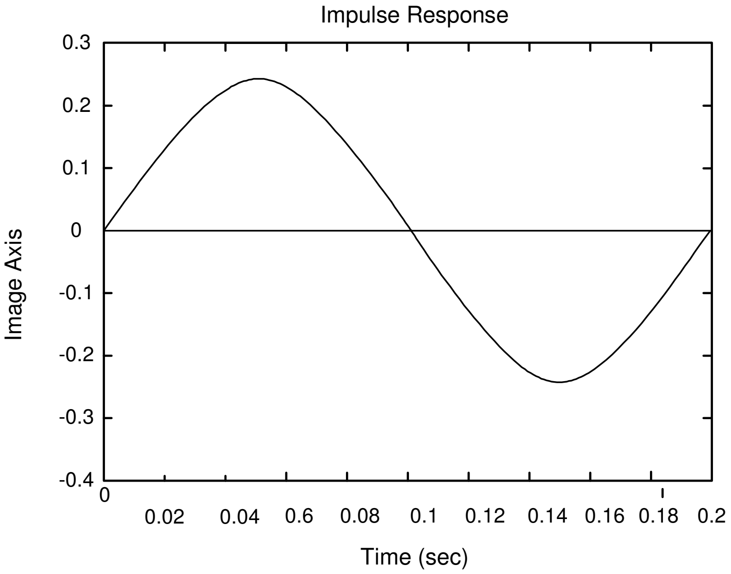
Step.3 Record x_0, i_0 and calculate c. The mass of the object,

$m = 10 \times 10^{-3}$ Kg (Specified by the manufacturer)
x_0 = 34 mm = 0.034 m
i_0 = 1.15 amp



$$G(s)=\frac{K.3922}{(s+569)(s^2-24^2)}$$

Fig. 8 Root Locus Diagram of the Uncompensated System



$$\frac{C}{R}(s)=\frac{G(s)}{1+G(s)}$$

Fig. 9 Impulse Reponse of the uncompensated System

$$c = mg\left(\frac{x_0}{i_0}\right)^2 = 10\times10^{-3}\times9.8\times\left(\frac{0.034}{1.15}\right)^2 = 8.56\times10^{-5}$$

Step.4 Compute K_0 and β (Ref to sec. 2.3) as,

$$K_0 = \frac{2ci_0}{mLx_0{}^2} = \frac{2g}{Li_0} = \frac{2\times9.8}{8.69\times10^{-3}\times1.15} =1961$$

$$\beta = \sqrt{\frac{2ci_0{}^2}{mx_0{}^3}} = \sqrt{\frac{2g}{x_0}} = \sqrt{\frac{2\times9.8}{0.034}} = 24.00$$

Step.5 The transfer function of the system at the nominal position is now written explicitly as

$$G(s) = \frac{2\times1961}{(s+569)(s^2 - 24.00^2)}$$ (Internal gain of power amplifier = 2)

The forward path transfer function may therefore be written as

$$K.G(s) = -\frac{K\times3922}{(s+569)(s^2-24.00^2)}$$

Where K is adjustable between 1 and 11 as explained in section 3.3

Step.6 Feedback path gain (sensor gain) at the nominal position is found by displacing the object slightly around the nominal position (say ±3mm) and monitoring the change in sensor output.

(i) $y_0 = 5$ mm Sensor output, $V_s = 3.62$ V

(ii) $y_0 + \Delta y = 8$ mm Sensor output, $V_s + \Delta V_s = 3.25$ V

(iii) $y_0 - \Delta y = 2$ mm Sensor output, $V_s - \Delta V_s = 4.63$ V

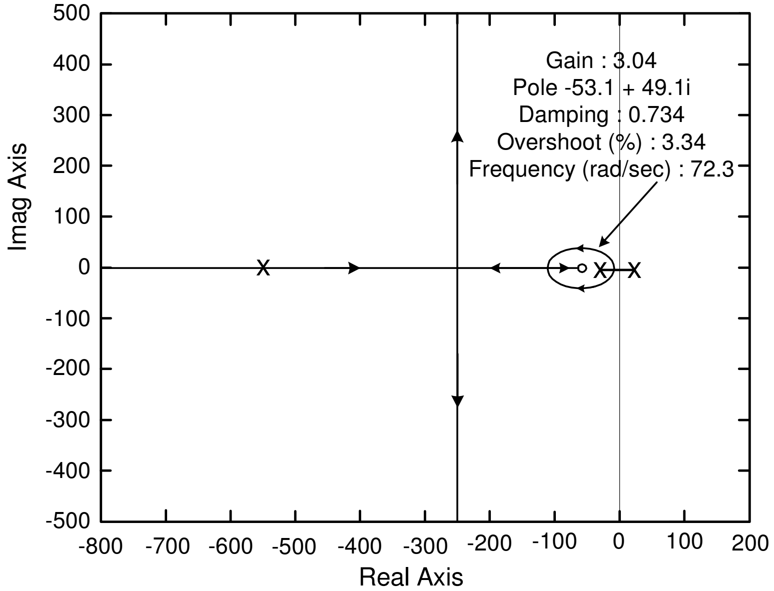
Average Sensor gain $K_s = \dfrac{2\Delta V_s}{2\Delta y_0} = 230$ V/m

The feedback path transfer function may therefore be written as

H(s) = 230

Step.7 Sketch the root locus diagram from the transfer function obtained in Step. 5 and 6 using MATLAB or otherwise, as shown Fig.8. Instability of the closed loop system for all values of open loop gain is obvious, and may also be verified through an impulse response plot as in Fig. 9.

Note that since the system has a non-unity feedback path transfer function, the root locus plot must take this into account. Sample MATLAB commands for this purpose are listed below.



$$GG_c = \frac{K\left(1+0.02s\right)\times 0.3922.230}{\left(s+569\right)\left(s^2-24^2\right)}$$

Fig. 10 Root Locus Diagram of the Compensated System

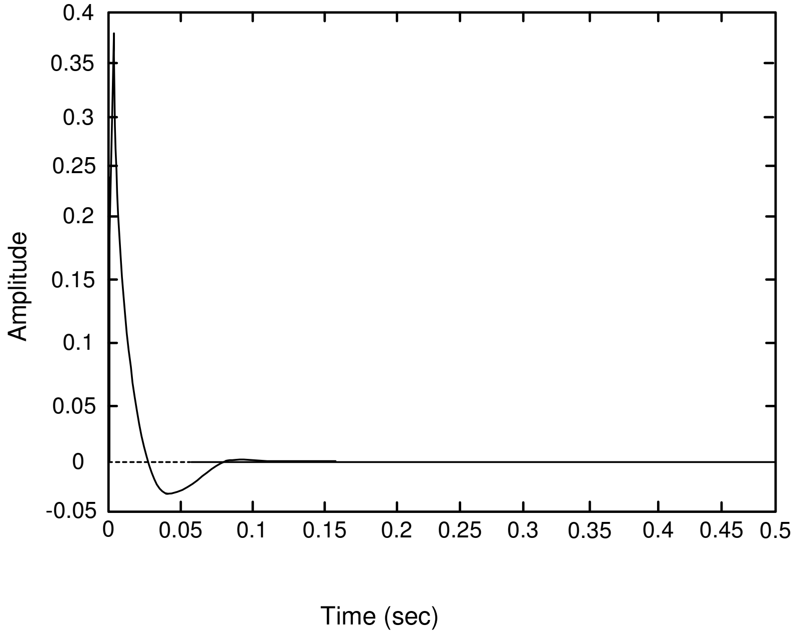


Fig. 11 Impulse Response of the closed Loop Compensated system with K=3

```
s=tf(`s’)
g=(3922)/((s+569)*(s∧2-24∧2));
h=230;
gh=g*h
rlocus(gh)
impulse(((g)/(1+gh)), .2)
```

Step.8 Choose a suitable closed loop pole location (P) and design a PD controller. A good choice may be a zero at S = - 50. The compensator transfer function. $G_C = (1+0.02s) = 0.02(s+50)$. With $R_1 = 10K\Omega$ (internal), the value of compensator capacitance is $C_1 = 2\mu F$. A revised root locus diagram may be drawn as shown in Fig. 10. Note that the system is now stable for K>1.5. Impulse response plot for K = 3 is shown in Fig. 11

MATLAB command:

```
gc=((1/50)*(s+50))
rlocus(gh*gc)
implus(((3*g*gc)/(1+3*gh*gc)), .0:.0001:.5)
```

Calculate and connect the designed value of capacitor on the panel and operate the system with the computed value of the forward path gain.

A careful observation of the root locus of Fig.10 would show that the system could be stabilized by using a wide range of capacitors. This may be verified experimentally.

Also it might appear that large values of K would be preferred in all cases. This however is not true in the practical situation due to the saturation of the amplifier.

5. FURTHER WORK

The experimental unit provides the user with an interesting platform to conduct further experimental work. This however is most conveniently done with the support of MATLAB. Some suggestions are:

(a) Simulate different PD controllers on MATLAB check the root locus diagram impulse response their effect on the unit.

(b) Attempt lead compensator design and implement it.

Note: Do not expect the experimental performance to match exactly with theoretical prediction. While the theoretical work is valid for linear system only, the experimental system is a non-linear one, which has been approximated as a linear system for small variation around the operating point.

6. REFERENCES

[1] Franklin GF, JD Powell and Michael Workman, “Digital Control of Dynamic System”, Addison Wesley, 2000.

[2] Shiao YS, “Design and Implementation of a controller for a Magnetic Leviation System”, Proc. Natl. Sci.Comc. ROC(D), Vol. II, No. 2, 2001, pp. 88-94.

Laboratory Manual

for

Control System Lab

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STUDY OF OPERATIONAL AMPLIFIER APPLICATIONS

OBJECT

To study the linear and nonlinear applications of a 741 operational amplifier as

(i) Integrator
(ii) Differentiator
(iii) Summer
(iv) Difference Amplifier
(v) Voltage to Current Converter
(vi) Current to Voltage Converter
(vii) Astable Mode of operation
(viii) Precision Rectifier

THEORY

An operational amplifier (OP-AMP) is a high gain direct-coupled amplifier having low output impedance and high input impedance. The first stage of an op-amp consists of a differential amplifier resulting in two input terminals called 'inverting' and 'Non-inverting' inputs. These are characterised by a -ve and +ve sign respectively. Since the amplifying stages are direct coupled and the nominal output is required to be at the ground potential, an op-amp operates from two power supplied, viz. $+V_{cc}$ and $-V_{cc}$. The input applied at 'Non inverting' terminal is amplified by the amplifier without any phase change whereas an input applied at 'Inverting' terminal undergoes a 180° phase change.

The operational Amplifier is a versatile device that can be used to amplify dc as well ac input signals and was originally designed for computing such mathematical functions as addition, subtraction, multiplication and integration. Thus the name operational amplifier stems from original use for these mathematical operations. With the addition of suitable external feedback components, the modern day op-amp can be used for a variety of applications such as integrator, differentiator, summer, subtractor, oscillators and others.

(a) Integrator: A circuit in which the output voltage waveform is the integral of the input voltage waveform is the integrator or the integration amplifier. Such a circuit is obtained by using a basic inverting amplifier configuration if the feedback resistor R_f is replaced by a capacitor C_f.

The expression for the output voltage V_o can be obtained by writing Kirchhoff's current equation at node V_2:

$$I_1 = I_b + I_f \qquad \ldots a(i)$$

Since I_b is negligibly small, $I_1 \approx I_f$

We know that the relationship between current through and voltage across the capacitor is

$$i_c = C\frac{dV_c}{dt}$$

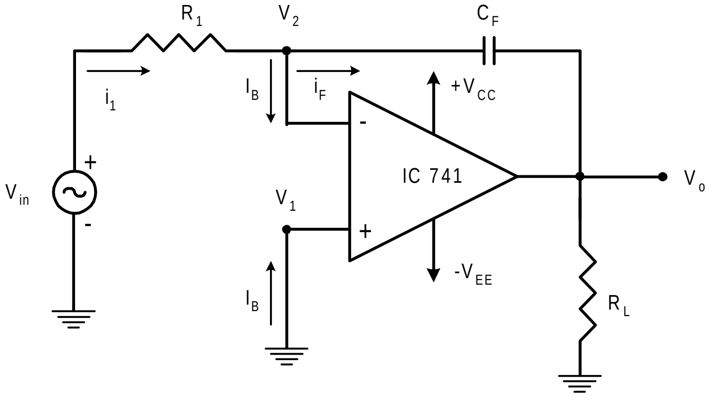

FIG. 1: Basic Integrator Circuit

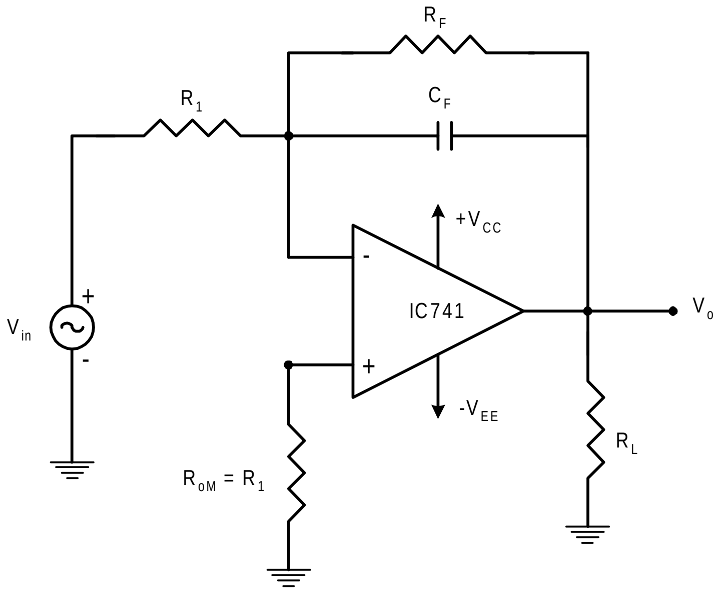

FIG. 2: A Practical Integrator

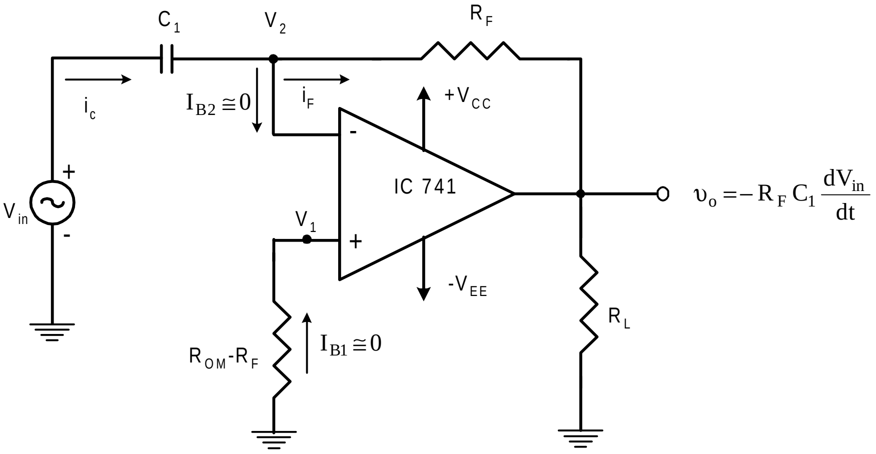

FIG. 3: Basic Differentiator Circuit

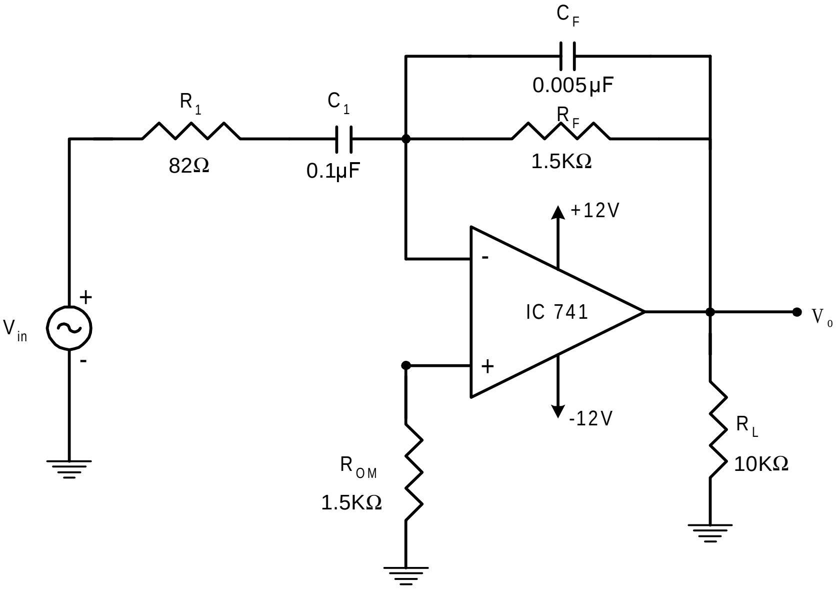

FIG. 4: A Practical Differentiator

Therefore, $\frac{V_{in}-V_2}{R_1}=C_F\left(\frac{d}{dt}(V_2-V_o)\right)$

However, V_1=V_2=0 because A is very large. Therefore,

$$\frac{V_{in}}{R_1}=-C_F\frac{dV_o}{dt}$$

The output voltage can be obtained by integrating both sides with respect to time:

$$\int_0^t \frac{V_{in}}{R_1}dt=\int_0^t C_F\frac{d}{dt}(-V_o)dt$$

$$=C_F(-V_o)+V_o\Big|_{t=0}$$

Therefore,

$$V_o=-\frac{1}{R_1C_F}\int_0^t V_{in}dt+C \qquad \ldots a(ii)$$

Where C is the integration constant and is proportional to the value of the output voltage V_o at time t=0 seconds. When V_{in} = 0, the integrator of fig. (1) works as an open loop amplifier, this is because the capacitor C_F acts as an open circuit ($X_{CF} = \infty$) to the input offset voltage V_{io}. In other words the input offset voltage V_{io} and the parts of the input current charging capacitor C_F produce the error voltage at the output of the integrator. Therefore, in the practical integrator shown in figure 2, to reduce the error voltage at the output, a resistor R_F is connected across the feedback capacitor C_F. Thus R_F limits the low frequency gain & hence minimizes the variation in the output voltage.

(b) Differentiator: As its name implies, the circuit performs the mathematical operation of differentiation; that is, the output waveform is derivative of the input waveform. The differentiator may be constructed from a basic inverting amplifier if an input resistor R_1 is replaced by capacitors $C_{1.}$

The expression for the output voltage can be obtained from Kirchhoff's current equation written at node V_2 as follows:

$$i_C = I_B + i_F \qquad \ldots b(i)$$

Since I_B= 0, $i_C = i_F$

$$C_1\frac{d}{dt}(V_{in}-V_2)=\frac{V_2-V_o}{R_f}$$

But V_1=V_2=0V (approx.), because A is very Large. Therefore,

$$C_1\frac{dV_{in}}{dt}=-\frac{V_o}{R_f}$$

$$V_o=-R_FC_1\frac{dV_{in}}{dt} \qquad \ldots b(ii)$$

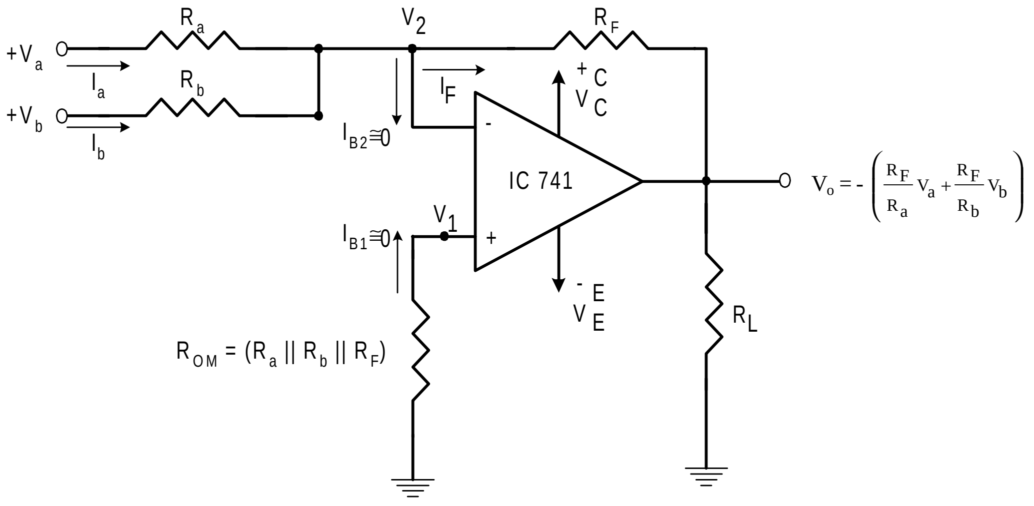

FIG. 5: Summing Amplifier (Inverting Configuration)

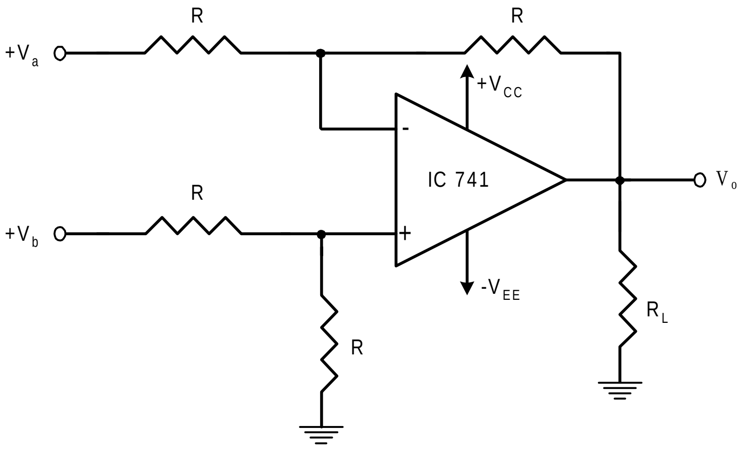

FIG. 6: Difference Amplifier

Thus the output V_o is equal to the R_FC_1 times the negative instantaneous rate of-change of the input voltage V_{in} with time. Since the differentiator performs the reverse of the integrator's function, a cosine wave input will produce a sine wave output, or a triangular input will produce a square wave output. However, the differentiator of Figure 3 will not do this because it has some practical problems. The gain of the circuit (R_FC_1) increases with increase in frequency at a rate of 20 dB/decade. This makes the circuit unstable. Also, the input impedance Xc_1 decreases with increase in frequency, which makes the circuit very susceptible to high-frequency noise. When amplified, this noise can completely override the differentiated output signal.

Both the stability and the high-frequency noise problems can be corrected by the addition of two components R_1 and C_F, as shown in Figure 4. This circuit is a *practical differentiator.*

(c) Summer Amplifier-Inverting Configuration: Figure 5 shows the inverting configuration with two inputs V_a, and V_b,. Depending on the relationship between the feedback resistor R_F and the input resistors R_a, and R_b, the circuit can be used as either a summing amplifier, scaling amplifier, or averaging amplifier. The circuit's function can be verified by examining the expression for the output voltage V_o, which is obtained from Kirchhoff’s current equation written at node V_2. Referring to Figure 5,

$$I_a + I_b = I_B + I_F \qquad \ldots c(i)$$

Since R_i and A of the op-amp are ideally infinity, I_B =0 A and $V_1=V_2=0V$(approx.), Therefore,

$$\frac{V_a}{R_a}+\frac{V_b}{R_b}=-\frac{V_o}{R_F}$$

$$V_o=-\left(\frac{R_F}{R_a}V_a+\frac{R_F}{R_b}V_b\right) \qquad \ldots c(ii)$$

Summing amplifier. If in the circuit of Figure 5, $R_a = R_b = R$, for example, then Equation c(ii) can be rewritten as

$$V_o=-\frac{R_F}{R_a}\left(V_a+V_b\right) \qquad \ldots c(iii)$$

This means that the output voltage is equal to the *negative* sum of all the inputs times the gain of the circuit R_F/R; hence the circuit is called a *summing amplifier.* Obviously, when the gain of the circuit is 1, that is, $R_a = R_b = R_F$, the output voltage is equal to the *negative* sum of all input voltages. Thus

$$V_o = -(V_a + V_b) \qquad \ldots c(iv)$$

(d) Difference Amplifier: A basic differential amplifier can be used as a *subtracter* as shown in Figure 6. In this figure, input signals can be scaled to the desired values by selecting appropriate values for the external resistors; when this is done, the circuit is referred to as *scaling amplifier.* However, in Figure 6, all external resistors are equal in value, so the gain of the amplifier is equal to 1.

From this figure, the output voltage of the differential amplifier with a gain of 1 is,

$$V_o=-\frac{R}{R}\left(V_a-V_b\right)$$

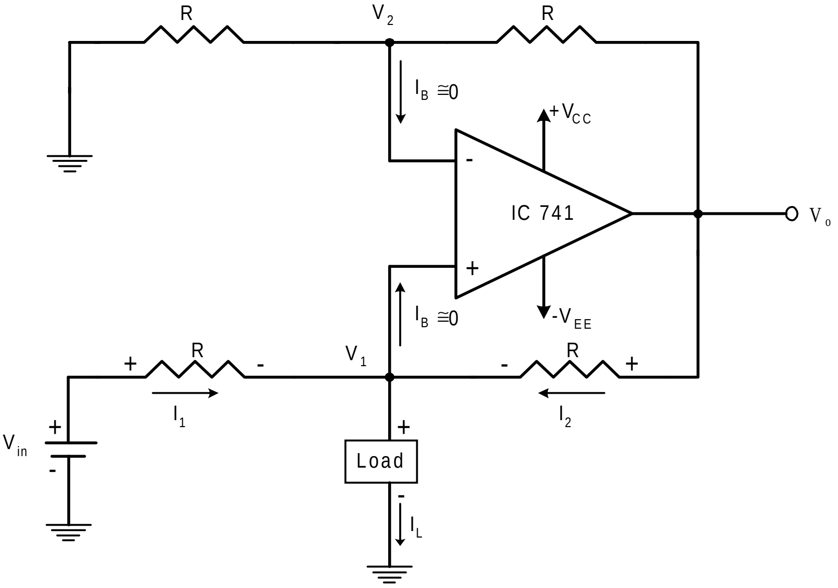

FIG. 7: Voltage to Current Converter

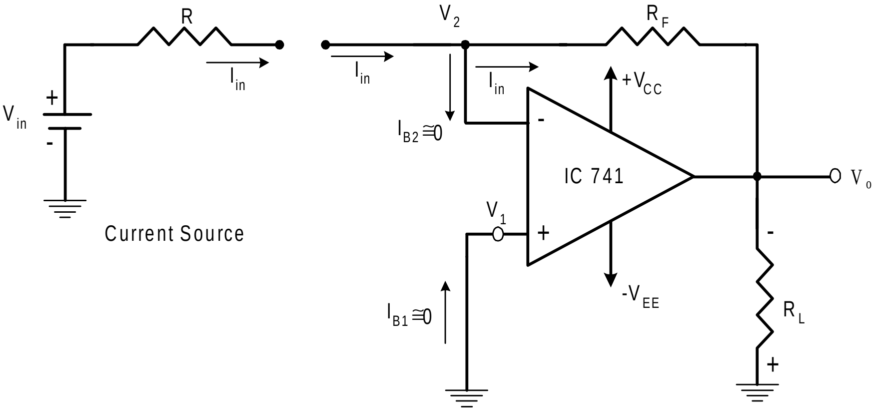

FIG. 8: Current to Voltage Converter

That is, $V_o = V_b - V_a$ …d(i)

Thus the output voltage V_o is equal to the voltage V_b, applied to the noninverting terminal *minus* the voltage V_a applied to the inverting terminal; hence the circuit is called a *subtracter*.

(e) Voltage to current converter: The voltage-to-current converter is shown in Figure 7. In this circuit, one terminal of the load is grounded, and an input voltage controls load current. The analysis of the circuit is accomplished by first determining the voltage V_1 at the noninverting input terminal and then establishing the relationship between V_1 and the load current.

Writing Kirchhoff's current equation at node V_1,

$$I_1 + I_2 = I_L$$

$$\frac{V_{in} - V_1}{R} + \frac{V_o - V_1}{R} = I_L$$

$$V_{in} + V_o - 2V_1 = I_L R$$

Therefore, $V_1 = \dfrac{V_{in} + V_o - I_L R}{2}$ …e(i)

Since the op-amp is connected in the noninverting mode, the gain of the circuit in figure 7 is 1+R/R=2. Then the output voltage is

$$V_o = 2V_1 \quad \text{…e(ii)}$$

$$= V_{in} + V_o - I_L R$$

That is, $V_{in} = I_L R$

or $I_L = V_{in} / R$ …e(iii)

This means that the load current depends on the input voltage V_{in} and resistor R and is independent of the 'Load'. Notice that all resistors must be equal in value.

(f) Current to Voltage converter: Let us consider the ideal voltage-gain equation of the inverting amplifier,

$$\frac{V_o}{V_{in}} = -\frac{R_F}{R_1}$$

Therefore,

$$V_o = -\left(\frac{V_{in}}{R_1}\right) R_F$$

However, since $V_1 = 0$ V and $V_1 = V_2$,

$$\left(\frac{V_{in}}{R_1}\right) = i_{in}$$

and $V_o = -i_{in} R_F$ …f(i)

This means that if we replace the V_{in} and R_1 combination by a current source I_{in} as shown in Figure 8, the output voltage V_o becomes proportional to the input current I_{in}. In other words, the circuit of Figure 8 *converts* the input current into a proportional output voltage.

One of the most common uses of the current-to-voltage converter is in sensing current from photo detectors and in digital-to-analog converter applications.

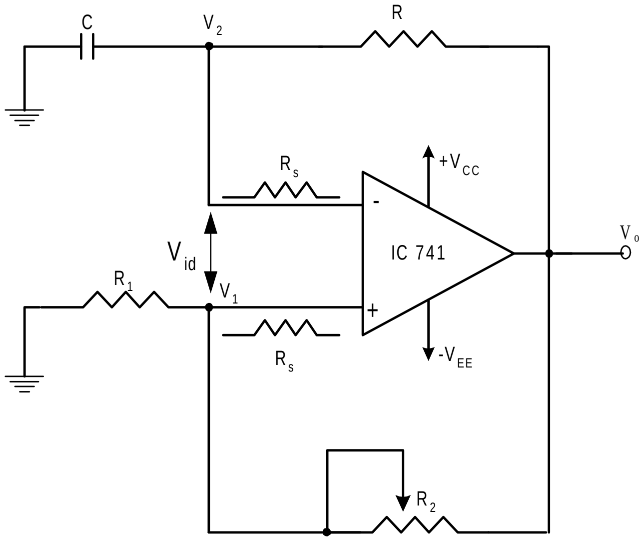

FIG. 9(a) Square Wave Generator

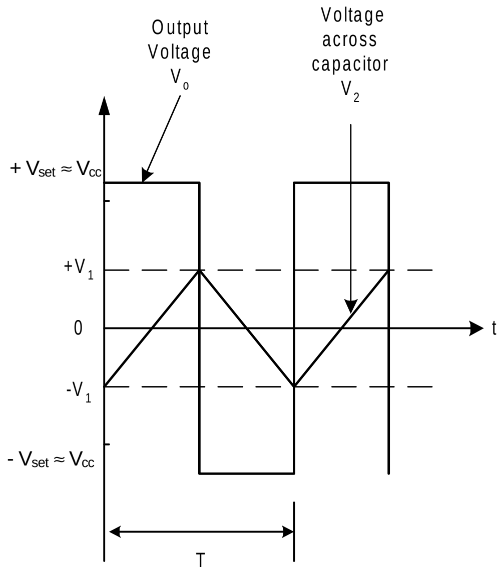

FIG. 9(b) Waveform of output voltage V_o and voltage across capacitor V_2.

(g) Astable Mode of operation or square wave generator: In contrast to sine wave oscillators, square wave outputs are generated when the op-amp is forced to operate in the saturated region. That is, the output of the op-amp is forced to swing repetitively between positive saturation $+V_{sat}$ ($\cong +V_{cc}$) and negative saturation $-V_{sat}$ ($\cong -V_{EE}$), resulting in the square-wave output. One such circuit is shown in Figure 9(a). This square wave generator is also called a *free running* or *astable* multivibrator. The output of the op-amp in this circuit will be in positive or negative saturation, depending on whether the differential voltage V_{id} is negative or positive, respectively.

Assume that the voltage across capacitor C is zero volts at the instant the dc supply voltages $+V_{cc}$ and $-V_{EE}$ are applied. This means that the voltage at the inverting terminal is zero initially. At the same instant, however, the voltage V_1 at the noninverting terminal is a very small finite value that is a function of the output-offset voltage V_{ooT} and the values of R_1 and R_2 resistors. Thus the differential input voltage V_{id} is equal to the voltage V_1 at the noninverting terminal. Although very small, voltage V_1 will start to drive the op-amp into saturation. For example, suppose that the output offset voltage V_{ooT} is also positive and that, therefore, voltage V_1 is also positive. Since initially the capacitor C acts as a short circuit, the gain of the op-amp is very large (A); hence V_1 drives the output of the op-amp to its positive saturation $+ V_{sat}$. With the output voltage of the op-amp at $+V_{sat}$, the capacitor C starts charging toward $+ V_{sat}$ through resistor R. However, as soon as the voltage v_2 across capacitor C is slightly more positive than V_1, the output of the op-amp is forced to switch to a negative saturation, $-V_{sat}$. With the op-amp's output voltage at negative saturation, $-V_{sat}$. The voltage v_1 across $R_1$is also negative, since

$$V_{LO} = \frac{R_1}{R_1+R_2}\left(-V_{sat}\right) \qquad \ldots g(i)$$

Thus the net differential voltage $V_{id} = V_1 - V_2$ is negative, which holds the output of the op-amp in negative saturation. The output remains in negative saturation until the capacitor C discharges and then recharges to a negative voltage slightly higher than $-V_1$. [See Figure 9(b).] Now, as soon as the capacitor's voltage V_2 becomes more negative than $-V_1$, the net differential voltage V_{id} becomes positive and hence drives the output of the op-amp back to its positive saturation $+V_{sat}$. This completes one cycle. With output at +Vsat voltage V_1 at the noninverting input is

$$V_1 = \frac{R_1}{R_1+R_2}\left(+V_{sat}\right) \qquad \ldots g(ii)$$

The time period T of the output waveform is given by

$$T = 2RC\ln\left(\frac{2R_1+R_2}{R_2}\right) \qquad \ldots g(iii)$$

or

$$f_0 = \frac{1}{2RC\ln\left[\left(2R_1+R_2\right)/R_2\right]} \qquad \ldots g(iv)$$

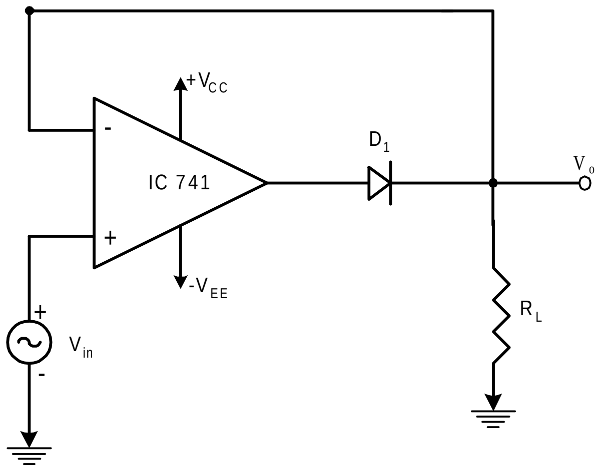

FIG. 10(a) Positive small signal half wave rectifier.

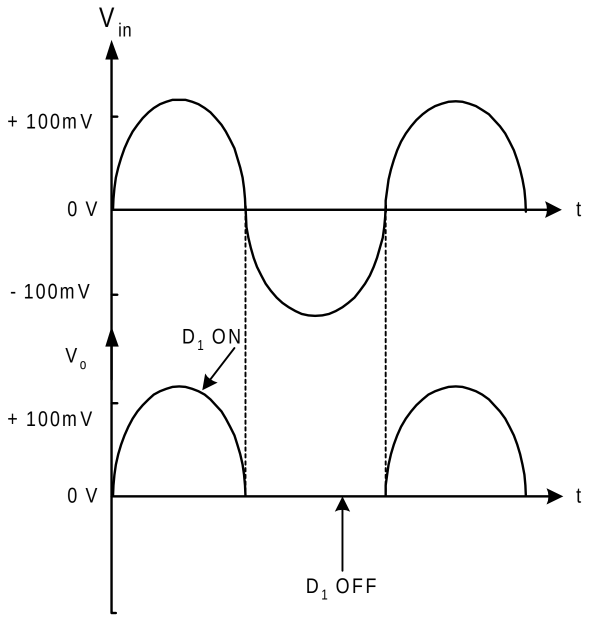

FIG. 10(b) Its output and input waveform

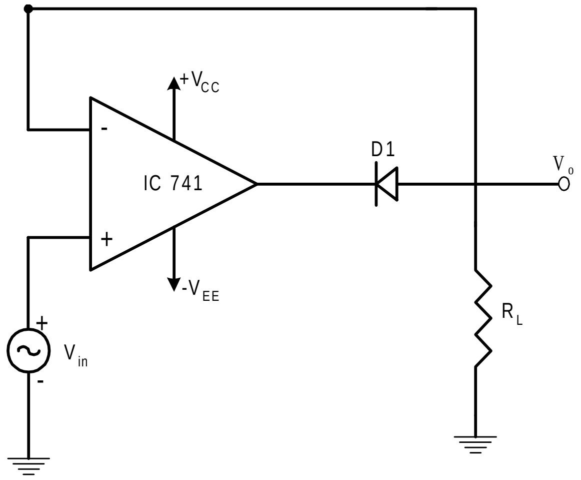

FIG. 11(a) Negative small signal half wave rectifier.

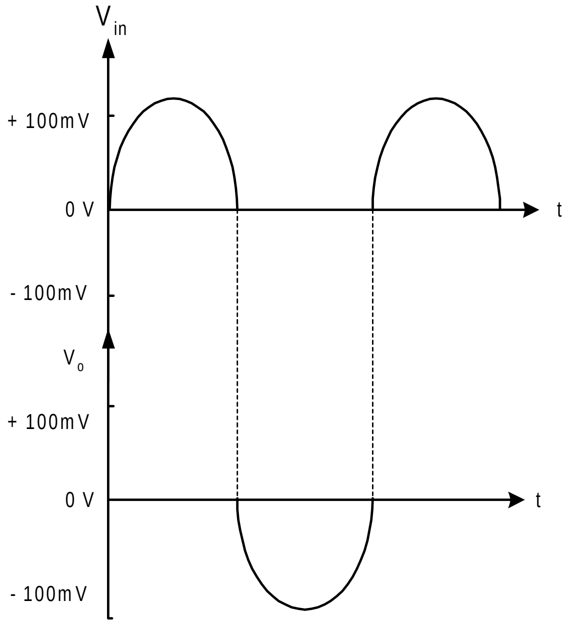

FIG. 11(b) Its output and input waveform

Equation g(iv) indicates that the frequency of the output f_o is not only a function of the RC time constant but also of the relationship between R_1 and R_2. For example, if R_2= l.l6R_1, Equation g(iv) becomes

$$f_o = \frac{1}{2RC} \qquad \ldots g(v)$$

Equation g(v) shows that the smaller the RC time constant, the higher the output frequency f_o, and vice versa. As with sine wave oscillators, the highest frequency generated by the square wave generator is also set by the slew rate of the op-amp. An attempt to operate the circuit at relatively higher frequencies causes the oscillator's output to become triangular. In practice, each inverting and noninverting terminal needs a series resistance R_s to prevent excessive differential current flow because the inputs of the op-amp are subjected to large differential voltages. The resistance R_s used should be 100 kΩ or higher. A reduced peak-to-peak output voltage swing can be obtained in the square wave generator of figure 9(a) by using back to back zeners at the output terminal.

(h) Precision Rectifier: The circuit of Figure 10(a) can be used as a positive small-signal half-wave rectifier provided that $-V_{ref} = 0$ V. The resultant circuit can rectify signals with peak values down to a few millivolts, unlike conventional diodes. This is possible because the high open-loop gain of the op-amp automatically adjusts the voltage drive to the diode $D_{1,}$ so that the rectified output peak is the same as the input [see Figure 10(b)]. In fact, the diode acts as an *ideal diode* (switch), since the voltage drop across the *on* diode is divided by the open-loop gain of the op-amp. As V_{in} starts increasing in the positive direction, the V_o' also starts increasing positively until diode D_1 is forward biased. When D_1 is forward biased, it closes a feedback loop and the op-amp works as a voltage follower. Therefore, the output voltage V_o follows the input voltage V_{in} during the positive half-cycle, as shown in Figure 10(b). However, when V_{in} starts increasing in the negative direction, V_o', also increases negatively until it is equal to the negative saturation voltage (= $-V_{EE}$). This reverse biases diode D_1 and opens the feedback loop. Therefore, during the negative half-cycle of the input signal, V_o is 0 V.

The op-amp in the circuit of Figure 10(a) must be a high-speed op-amp since it alternates between open-loop and closed-loop operations. µA318, HA2500, and LM310 are typical examples of high-speed op-amps. However, in the present set-up 741 is used instead although at low frequencies.

Figure 11(a) shows a negative small-signal half-wave rectifier. During the positive alternation of V_{in}, D_1 is reverse biased; therefore, V_o = 0V. On the other hand, during the negative alternation, D_1 is forward biased; hence V_o follows $V_{in.}$

Yet another negative half-wave rectifier is shown in Figure 12. In this circuit two diodes are used so that the output V_o' of the op-amp does not saturate. This minimizes the response time and increases the operating frequency range of the op-amp. However, notice that the op-amp is used in the inverting configuration, and the output is measured at the anode of diode D_1 with respect to ground.

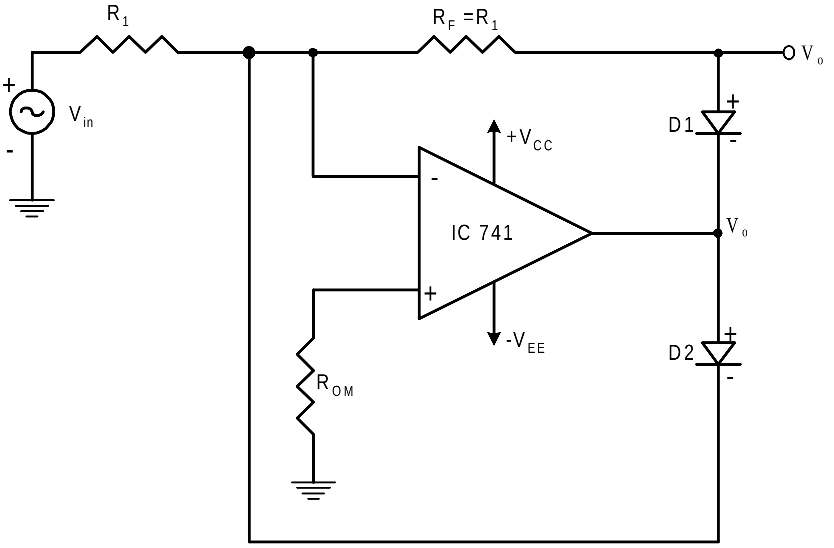

FIG. 12 Negative half wave rectifier.

PROCEDURE

(a) Integrator:

(i) Make the connections as shown in figure 2.

(ii) Set R_1=10KΩ, R_F= 10KΩ, R_{om}=10 KΩ, C_F=0.1μF.

(iii) Apply the square wave input voltage V_{in} (in-built) and observe its amplitude and frequency on CRO.

(iv) Switch ON the unit and observe the output (triangular wave) amplitude and frequency on CRO.

(v) Tabulate the amplitude and frequency of input and output by varying the frequency of the input by frequency pot.

(vi) Verify the results with the help of equation a(ii).

(b) Differentiator:

(i) Make the connections as shown in figure 4.

(ii) Set R_1=270Ω, R_F= 100KΩ, R_{om}=10 KΩ, C_1=0.1μF.

(iii) Apply the triangular wave input voltage V_{in} (in-built) and observe its amplitude and frequency on CRO. (Ensure that switch S_1 is on triangular position)

(iv) Switch ON the unit and observe the output (square wave) amplitude and frequency on CRO.

(v) Tabulate the amplitude and frequency of input and output by varying the frequency of the input by frequency pot.

(vi) Verify the results with the help of equation b(ii).

(c) Summer Amplifier:

(i) Make the connections as shown in figure 5.

(ii) Set R_a =R_b= R_F = R_{om}=10 KΩ

(iii) Apply the voltage V_a and V_b with the help of inbuilt voltage sources and measure its amplitude with the help of DVM.

(iv) Switch ON the unit and measure the output voltage on DVM.

(v) Tabulate the V_a, V_b, V_o.

(vi) Verify the results with the help of equation C(iv)

(vii) Set the different values of R_a, R_b, R_f, R_{om} and repeat the steps (iii) to (vi).

(d) Difference Amplifier:

(i) Make the connections as shown in figure 6.

(ii) Set R =10 KΩ

(iii) Apply the voltage V_a and V_b with the help of inbuilt voltage sources and measure its amplitude with the help of DVM.

(iv) Switch ON the unit and measure the output voltage on DVM.

(v) Tabulate the V_a, V_b, V_o.

(vi) Verify the results with the help of equation d(i)

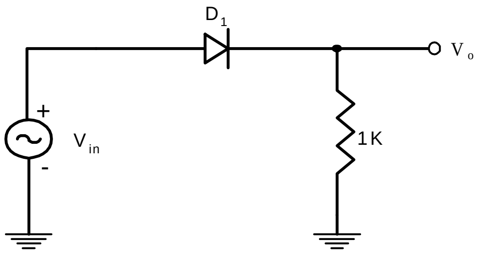

FIG. 13(a) Normal Rectifier

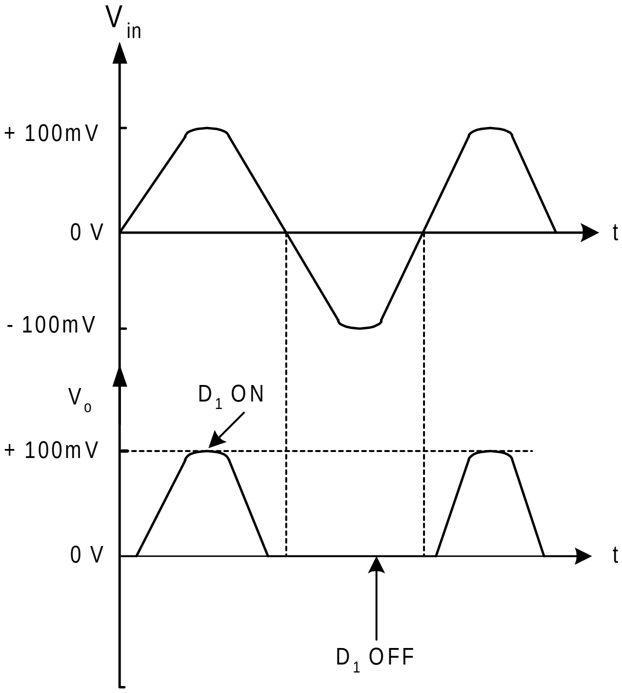

FIG. 13(b) Its input & output waveform

(e) Voltage to current converter:

(i) Make the connections as shown in figure 7.

(ii) For the voltage input (V_{in}) use in-built voltage source.

(iii) Set R=10 KΩ

(iv) Tabulate the load current on different loads

(f) Current to voltage converter:

(i) Make the connections as shown in figure 8.

(ii) Set R_F =270Ω

(iii) Measure the output voltage $V_{o.}$

(iv) Use different values of R_F upto 10KΩ.

(v) Tabulate the V_o and $R_{F.}$

(vi) Verify the expression shown in equation f(i).

(g) Astable Mode of operation:

(i) Make the connections as shown in figure 9(a)

(ii) Set R_1 =10 KΩ, R_2=10 KΩ, R_F = 10 KΩ, C=0.047μF

(iii) Switch ON the unit and observe the output square wave and voltage across capacitor on CRO and measure its frequency

(iv) Calculate the theoretical frequency with the help of equation g(iv) and verify the results

(v) Set the different values of R_1, R_2, R_f, and C and repeat the steps (iii) to (iv)

(h) Precision Rectifier: In that experiment first of all we will see the response of normal rectifier as follows:

(i) Make the connections as shown in figure 13(a).

(ii) Apply the sine wave input $V_{in} \approx$ 1V (p-p) (in-built) and observe that wave form on CRO. (Ensure that the switch S_1 is on sine wave position)

(iii) Switch ON the unit and observe the output wave form on CRO and trace it.

Now we will see the response of precision rectifier

(i) Make the connections as shown in figure 10(a).

(ii) Apply the sine wave input $V_{in} \approx$ 1V (p-p) (in-built) and observe that waveform on CRO. (Ensure that the switch S_1 is on sine wave position)

(iii) Switch ON the unit and observe the output wave form on CRO and trace it.

(iv) Compare that output voltage trace with the normal diode output voltage trace.

NOTE: Overflow in the DVM is indicated by a steady reading of '1' In such case either switch the meter to the higher range or decrease the input.

TYPICAL RESULTS

a) INTEGRATOR:

Input V_{in} (Square wave) = 5.0 V(p-p), 1KHz

Output V_o (Triangular) = 1.25 V(p-p), 1KHz

Calculations:- $V_o = - \frac{1}{R_1 C_F} \int_0^t V_{in}\, dt$

$$= - \frac{1}{10 \text{x} 10^3 \text{ x} 0.1 \text{x} 10^{-6}} \int_0^{0.5 \text{x} 10^{-3}} 2.5 dt$$

$$= - \frac{1}{10^{-3}} \left[2.5 \left(0.5 \text{x} 10^{-3} - 0 \right) \right]$$

$$= - \frac{1}{10^{-3}} \left[2.5 \text{x} 0.5 \text{x} 10^{-3} \right] = - 1.25 \text{ V}$$

Slope = $2.5 \text{ x } 10^3$ V/sec

Result: Equation a(ii) is verified and the slope of the integrator is $2.5 \text{ x } 10^3$ V/sec

b) Differentiator:

Input V_{in} (Triangular wave) = 2.5 V(p-p), 1KHz

Output V_o (Square wave) = 10 V(p-p), 1KHz

Calculations:- $V_o = - R_f C_1 \frac{dV_{in}}{dt}$

$$= - 10 \text{ x } 10^3 \text{ x } 0.1 \text{ x } 10^{-6} \frac{2.5}{0.25 \text{x} 10^{-3}}$$

$$= - 10 \text{ V}$$

Result: Equation b(ii) is verified

c) Summer: $R_a = R_b = R_f = R_{om}$ = 10KΩ

S. No.	(V_a)	(V_b)	(V_o) Measured	(V_o) Theoritical
1.	0.503V	0.502V	- 1.004V	- 1.005V
2.	0. 503V	0.999V	- 1.500V	- 1.502V
3.	1.000V	1.000V	- 2.000V	- 2.000V
4.	- 1.000V	1.000V	0.000V	0.000V
5.	1.000V	- 0.500V	- 0.500V	- 0.500V

Result: Equation c(iv) is verified.

d) Difference Amplifier:

S. No.	(V_a)	(V_b)	(V_o) Measured	(V_o) Theoritical
1.	- 0.50V	1.00V	1.51V	1.50V
2.	- 1. 00V	1.00V	2.02V	2.00V
3.	0.50V	1.00V	0.50V	0.50V
4.	1.00V	1.00V	0.00V	0.00V
5.	1.00V	0.50V	- 0.50V	- 0.50V
6.	1.00V	- 1.00V	- 2.01V	- 2.00V

Result: Equation d(i) is verified.

e) Voltage to Current converter:

$V_{in} = + 1V,$ $V_1 = 0.10V$

$V_o = 0.20V,$ $R = 10K\Omega$

$$I_L = \frac{V_{in}}{R}$$

$$= \frac{1}{10K} = 0.1mA$$

and $V_o = 2\ V_1$

Result: Equation e(ii) is verified.

f) Current to Voltage converter:

$I_{in} = \frac{1V}{1K} = 1mA$ and $V_o = -\ i_{in} R_f$

S.No.	R_f	(V_o) Measured	(V_o) Theoritical
1.	270 E	- 0.26V	- 0.27V
2.	1K	- 0.98V	- 1.00V
3.	10K	- 9.67V	- 10.00V

Result: Equation f(i) is verified.

g) Astable Mode of Operation:

$R = 10.083\ K\Omega$

$R_1 = 10.120\ K\Omega$

$R_2 = 10.133\ K\Omega$

$C = 0.049\ \mu F$

$$f_{o(\text{Measured})} = \frac{1}{1.1}\ \text{KHz} \cong 909\ \text{Hz}$$

$$f_{o(\text{Theoritical})} = \frac{1}{2RC\ln\left[\frac{(2R_1 + R_2)}{R_2}\right]}$$

$$= \frac{1}{2\,x\,10.083\,x\,10^3\ x\,0.049\,x\,10^{-6}\ \ln\left[\frac{2\,x\,10.120 + 10.133}{10.133}\right]}$$

$$= \frac{1}{0.988\,x\,10^{-3}\ \ln[2.997]}$$

$$= 921\ \text{Hz}$$

Result: $f_{o(\text{Measured})} \cong f_{o(\text{Theoritical})}$

h) Precision Rectifier:

V_{in} = 1.2V(p-p) [+ 0.6V to –0.6V]

With Normal Rectifier:

$V_{out}|_{max}$ = 0.3V

With Precision Rectifier:

$V_{out}|_{max}$ = 0.6V

Result: We can observe that the precision rectifier rectifies more precisely with respect to the normal rectifier.

NOTE: All the values of capacitors mounted on panel are with in ± 5%. So far accurate results, measure these values by 4½ digit LC meter.

Laboratory Manual

for

Control System Lab

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STROBOSCOPE STB-01

1. OVERVIEW

The STROBOSCOPE is an instrument designed for the visually stopping the motion in rotating shaft for diagnostic inspection purposes and for measurement of shaft speed in revolutions per minute (RPM). The instrument has applications in a variety of applications in industrial and laboratory environment. A principal feature of the STROBOSCOPE is its non-contact nature, which enables the user to monitor the shaft speed from a distance. Also, the speed of very small motors may be measured without any loading errors. The speed is displayed on a 4-digit readout from 500-9900rpm (guaranteed range). A 10-turn potentiometer is used to span the whole range of speed very conveniently. The instrument operates from the 220V, 50 Hz line while all internal supplies are IC regulated.

2. STROBOSCOPE SCHEMATIC Diagram

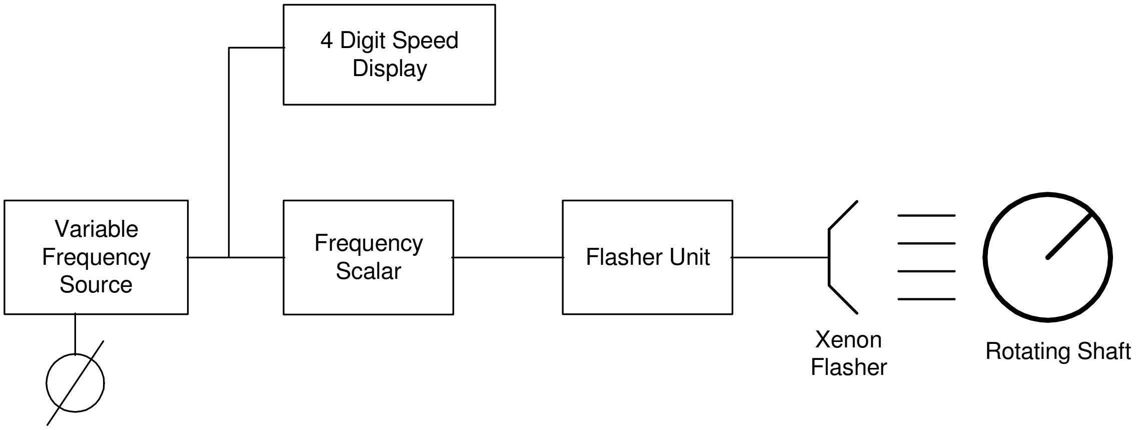

Fig. 2 Schematic Diagram

3. PRINCIPLE OF OPERATION

In a stroboscope high intensity light flashes are directed towards a rotating shaft on which a distinct marking has already been drawn or pasted. The period of the flashes may be varied manually. The marking would appear stationary when the time for one shaft revolution equals the flash period. This is the fundamental period. Also single stationary images will be seen if the flash period is an integral multiple of the above value. However for sub multiple flash periods multiple stationary images will be seen. The fundamental period is used for speed measurement and this is identified as the smallest period (highest frequency) for which a single stationary image is seen.

Referring to the circuit diagram shown, a variable frequency square wave oscillator supplies the timing pulses. These are used to periodically trigger a flash tube after suitable scaling, so as to correspond to speed reading in r.p.m. The power supply unit supplies power to various subsystems. Speed is read on a 4-digit display of the r.p.m. counter having a crystal controlled window for greater accuracy.

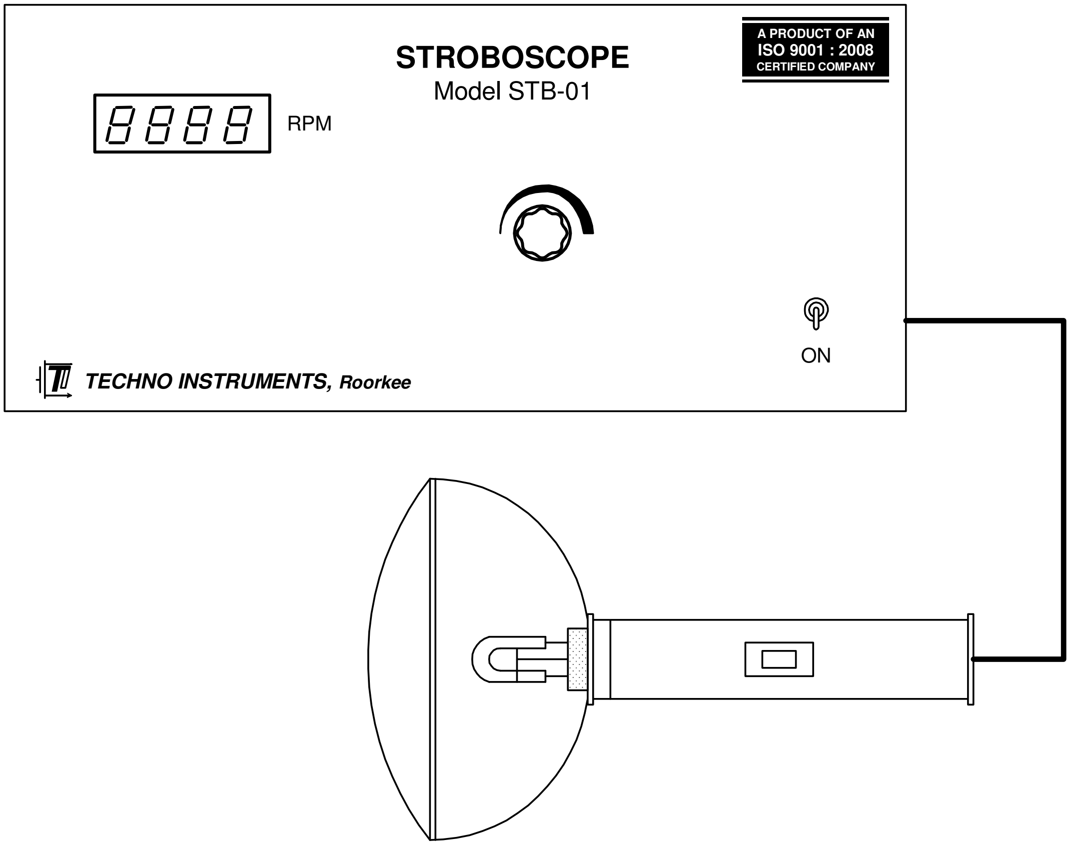

Fig. 1 Panel Drawing Stroboscope, STB-01

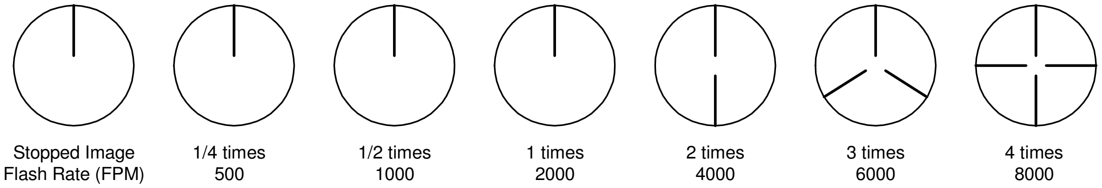

Fig. 3 Object Rotating at 2000 rpm

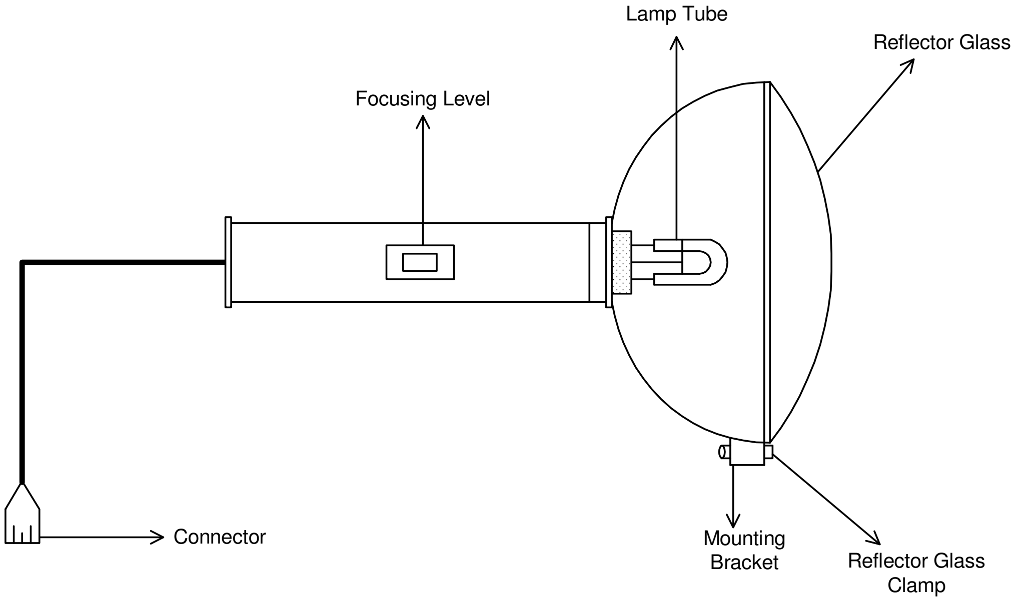

Fig. 4 Stroboscope Flasher Unit

Safeguards and Precautions

1. Read and follow all instructions in this manual carefully, and retain this manual for future reference.
2. Do not use this instrument in any manner inconsistent with these operating instructions or under any conditions that exceed the environmental specifications stated.
3. Use of this product may induce and epileptic seizure in person prone to this type of attack.
4. Objects viewed with this product may appear to be stationary when in fact they are moving at high speeds. Always keep a safe distance from moving machinery and do no touch the target.
5. There are lethal voltages present inside this product. Refer to the section on Lamp Replacement before attempting to open this product.
6. Do not allow liquids or metallic objects to enter the ventilation holes on the stroboscope as this may cause permanent damage and void the warranty.
7. Do not allow cables extending from unit to come into contact with rotating machinery, as serious damage to the equipment, or severe personal injury or death may occur as a result.
8. Do not direct strobe flash toward certain data collectors, as it may temporarily interrupt data collector operation, and could result in loss of stored data.
9. This instrument is not user serviceable. For technical assistance, contact the sales organization from which you purchased the product or Techno Instruments Instrument directly.

4. OPERATING PROCEDURE

4.1 Using Stroboscope for diagnostic purpose

(a) Connect the connector of lamp unit to the socket given at the back of main unit

(b) Switch 'ON' the STROBOSCOPE and direct the flashes at point of inspection on the rotating shaft.

(c) For focusing the light on the object, unscrew slightly the screw given on top of focusing lever, given at the side of Lamp. Adjust the lever to obtain best focus of the light on the object. Once the desired focus is achieved tighten the screw.

(d) Vary the knob on the panel of the main unit to attain a stationary image of the shaft.

4.2 Given below are the steps for reading the speed of a rotating shaft.

(a) While the shaft is stationary, put a mark or a sticker on it.

(b) Allow the shaft to rotate.

(c) Connect the connector of lamp unit to the socket given at the back of main unit

(d) Switch 'ON' the STROBOSCOPE and direct the flashes at the mark on the shaft.

(e) If focusing is required, unscrew slightly the screw given on top of focusing lever, given at the side of Lamp. Adjust the lever to obtain best focus of the light on the object. Once the desired focus is achieved tighten the screw.

(f) For measurement, starting from the high speed end (extreme clock wise) turn the 'SPEED' potentiometer down to lower speed till a single stationary image is observed.

(g) The reading on the display of Stroboscope directly gives speed in r.p.m.

(h) To confirm the true speed, note the reading and adjust the stroboscope to exactly half of this reading. You should again see a single image (which may be phase shifted with respect to the first image)

(i) If the speed is outside the full rage of the stroboscope (9900 rpm), it can be measured using the method of harmonics and multipoint calculation. Start at the highest rate and adjust the flash rate down. Be aware that you will encounter multiple images. Note the flash rate of the first SINGLE image you encounter and call this speed "A". Continue decreasing the flash rate until you encounter a second SINGLE image; note this speed as "B". Continue decreasing the speed until you reach a third SINGLE image at speed "C".

For a two point calculation the actual speed is given by

$$RPM = AB/ (A+B)$$

For a three point calculation

$$RPM = 2XY(X+Y)/ (X-Y)^2$$ where

$X = (A-B)$ and

$Y = (B-C)$

4.3 Lamp & Fuse Replacement

Warning: Before attempting to remove the lamp, make sure the stroboscope is turned off and any mains cord is removed from the AC outlet. Allow the lamp to cool waiting for atleast 1 minute.

The stroboscope is designed to discharge the internal high voltage within 30 seconds. However caution should be exercised when replacing the lamp.

4.3.1 To change the lamp:

1. Unscrew the clamp holding the reflector glass on the lamp unit.
2. Carefully remove the clamp and the reflector glass.
3. Hold the lamp with a cloth between your forefingers and thumb and rock it back and forth gently while pulling out. Do not attempt to rotate the lamp. The lamp is socketed and will come out easily when pulled.
4. The lamps are polarized and must be put into socket matching polarity. Using a lint free cloth, match the metal ring on the tube with the black ring mark on the socket and gently rock the lamp back and forth while pushing it into place (as shown in the figure). Make sure the lamp is straight and centered in the reflector hole.

   Warning: Do not allow the reflector to come in contact of the lamp.
5. Reinstall the front glass and check that the clamp screw is sufficiently so as to firmly secure the front glass.

4.3.2 Fuse Replacement

There is a cartage type fuse inside the main unit which may be accessed by removing the two screws given on the top panel of the main unit. Under normal operating conditions, the fuse should never blow. Examples of abnormal operating conditions would be foreign material entering the strobo (main or lamp unit), such as water, ink etc. If the fuse needs to be replaced, replace only with a fuse of the same type and value

Fuse: 3/4 “cartage type, Value: 200mA

Technical specifications

(i) Speed Range: 300 - 9900 r.p.m.

(ii) Normal Flash Energy for high FPS shot : 0.02 Joule/Sec

(iii) Average Power Input Max. of the lamp: 5 watt

(iv) Average Flash Life of the lamp tube: 100Hrs.

(v) Reading accuracy : better than ± 0.02% ± 1 digit

(vi) Viewing distance : 0.5 - 5m depending on ambient light

(vii) Operation : Continuous

(viii) Power : 220V, 50 Hz (nominal)

Laboratory Manual

for

Control System Lab

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Software Experiment 1: Linear Time-invariant Systems and Representation

Objectives: This experiment has following two objectives:

1. Continued with the learning of Mathematical Modeling from previous experiment, we now start focusing the linear systems. We will learn commands in MATLAB that would be used to represent such systems in terms of transfer function or pole-zero-gain representations.
2. We will also learn how to make preliminary analysis of such systems using plots of poles and zeros locations as well as time response due to impulse, step and arbitrary inputs.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft-Experiment

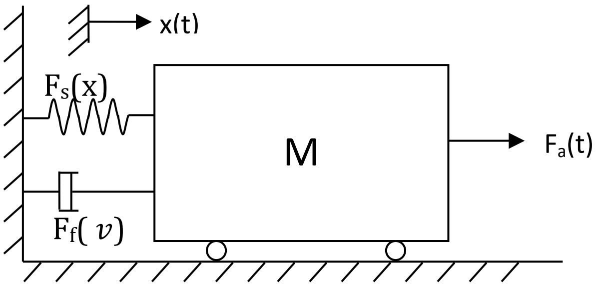
Deliverables

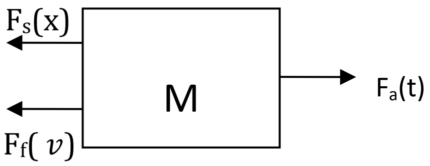
A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results should be reported properly.

Mass-Spring System Model

The spring force is assumed to be either linear or can be approximated by a linear function $F_s(x)= Kx$, B is the friction coefficient, x(t) is the displacement and $F_a(t)$ is the applied force:





The differential equation for the above Mass-Spring system can be derived as follows

$$M\frac{d\ \ x(t)}{dt}+B\frac{dx(t)}{dt}+Kx(t)=F_a(t)$$

Transfer Function:

Applying the Laplace transformation while assuming the initial conditions are zeros, we get

$$(Ms\quad+Bs+K)*X(s)=F_a(s)$$

Then the transfer function representation of the system is given by

$$TF=\frac{Output}{Input}=\frac{F_a(s)}{X(s)}=\frac{1}{(Ms\quad+Bs+K)}$$

Linear Time-Invariant Systems in MATLAB:

Control System Toolbox in MATLAB offers extensive tools to manipulate and analyze linear time-invariant (LTI) models. It supports both continuous- and discrete-time systems. Systems can be single-input/single-output (SISO) or multiple-input/multiple-output (MIMO). You can specify LTI models as:

Transfer functions (TF), for example,

$$P(s)=\frac{s+2}{s\quad+s+10}$$

<u>**Note:**</u> All LTI models are represented as a ratio of polynomial functions

Examples of Creating LTI Models

Building LTI models with Control System Toolbox is straightforward. The following sections show simple examples. Note that all LTI models, i.e. TF, ZPK and SS are also MATLAB objects.

Example of Creating Transfer Function Models

You can create transfer function (TF) models by specifying numerator and denominator coefficients. For example,

```
>>num = [1 0];
>>den = [1 2 1];
>>sys = tf(num,den)
```

```
Transfer function:
     s
-------------
s^2 + 2 s + 1
```

A useful trick is to create the Laplace variable, s. That way, you can specify polynomials using s as the polynomial variable.

```
>>s=tf('s');
>>sys= s/(s^2 + 2*s + 1)
```

```
Transfer function:
    s
-------------
s^2 + 2 s + 1
```

This is identical to the previous transfer function.

Example of Creating Zero-Pole-Gain Models

To create zero-pole-gain (ZPK) models, you must specify each of the three components in vector format. For example,

```
>>sys = zpk([0],[-1 -1],[1])
```

```
Zero/pole/gain:

s
-------
(s+1)^2
```

produces the same transfer function built in the TF example, but the representation is now ZPK. This example shows a more complicated ZPK model.

```
>>sys=zpk([1 0], [-1 -3 -.28],[.776])
```

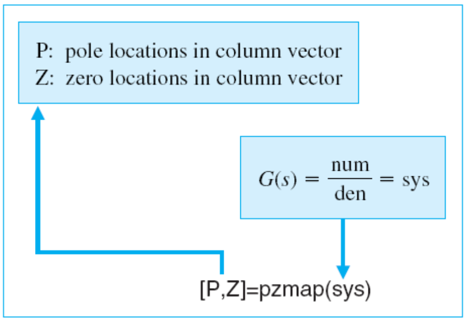
```
Zero/pole/gain:
  0.776 s (s-1)
--------------------
(s+1) (s+3) (s+0.28)
```

Plotting poles and zeros of a system:

pzmap

Compute pole-zero map of LTI models

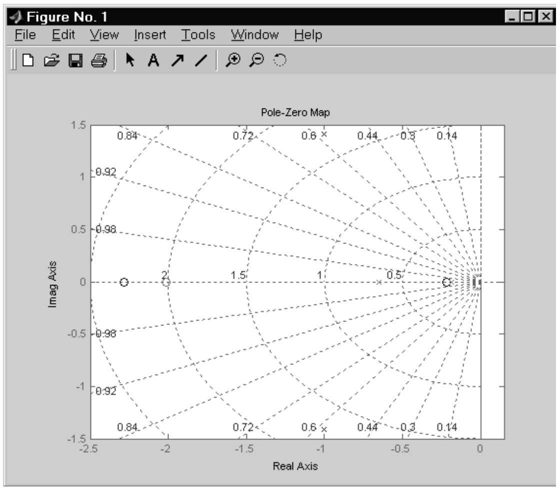
```
pzmap(sys)
pzmap(sys1,sys2,...,sysN)
[p,z] = pzmap(sys)
```



Description:

pzmap(sys) plots the pole-zero map of the continuous- or discrete-time LTI model sys. For SISO systems, pzmap plots the transfer function poles and zeros. The poles are plotted as x's and the zeros are plotted as o's.

pzmap(sys1,sys2,...,sysN) plots the pole-zero map of several LTI models on a single figure. The LTI models can have different numbers of inputs and outputs. When invoked with left-hand arguments,



[p,z] = pzmap(sys) returns the system poles and zeros in the column vectors p and z. No plot is drawn on the screen. You can use the functions sgrid or zgrid to plot lines of constant damping ratio and natural frequency in the s- or z- plane.

Example

Plot the poles and zeros of the continuous-time system.

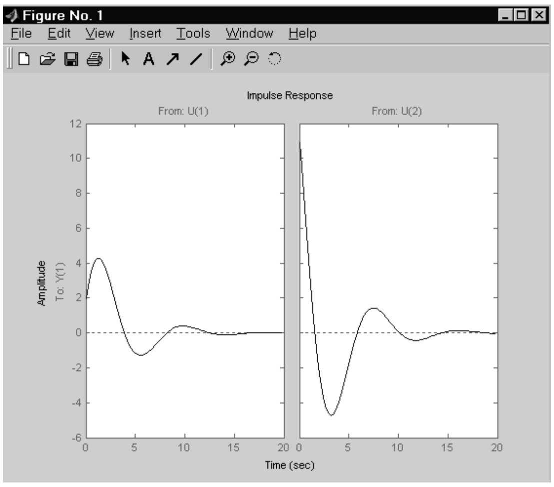
$$H(s) = \frac{2s \quad + 5s + 1}{s \quad + 2s + 3}$$

```
>>H = tf([2 5 1],[1 2 3]); sgrid
>>pzmap(H)
```

Simulation of Linear systems to different inputs

impulse, step and lsim

You can simulate the LTI systems to inputs like impulse, step and other standard inputs and see the plot of the response in the figure window. MATLAB command ‘impulse’ calculates the unit impulse response of the system, ‘step’ calculates the unit step response of the system and ‘lsim’ simulates the (time) response of continuous or discrete linear systems to arbitrary inputs. When invoked without left-hand arguments, all three commands plots the response on the screen. For example:



To obtain an impulse response

```
>> H = tf([2 5 1],[1 2 3]);
>>impulse(H)
```

To obtain a step response type

```
>>step(H)
```

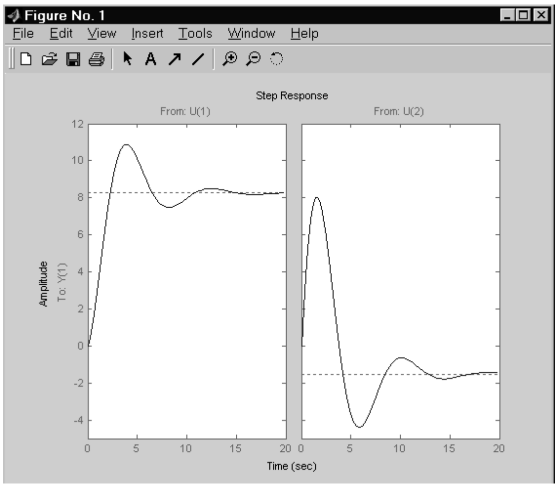
Time-interval specification:

To contain the response of the system you can also specify the time interval to simulate the system to. For example,

```
>> t = 0:0.01:10;
>> impulse(H,t)
```

Or

```
>> step(H,t)
```



Simulation to Arbitrary Inputs:

To simulates the (time) response of continuous or discrete linear systems to arbitrary inputs use ‘lsim’. When invoked without left-hand arguments, ‘lsim’ plots the response on the screen.

lsim(sys,u,t) produces a plot of the time response of the LTI model sys to the input time history ‘t’,’u’. The vector ‘t’ specifies the time samples for the simulation and consists of regularly spaced time samples.

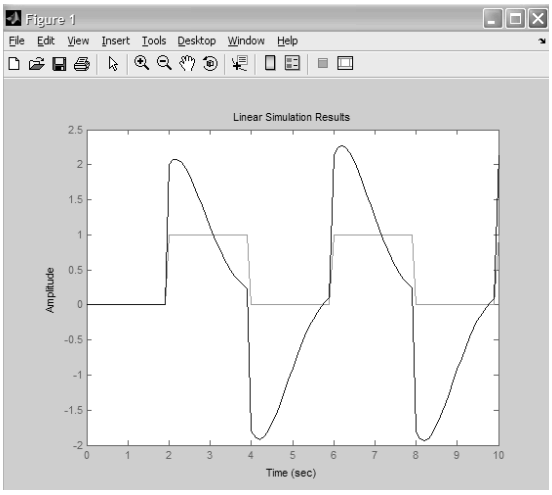
T = 0:dt:Tfinal

The matrix u must have as many rows as time samples (length(t)) and as many columns as system inputs. Each row u(I,☺ specifies the input value(s) at the time sample t(i).

Simulate and plot the response of the system

$$H(s) = \frac{2s \quad + 5s + 1}{s \quad + 2s + 3}$$

to a square wave with period of four seconds.



First generate the square wave with gensig. Sample every 0.1 second during 10 seconds:

```
>>[u,t] = gensig(‘square’,4,10,0.1);
```

Then simulate with lsim.

```
>> H = tf([2 5 1],[1 2 3])
```

```
Transfer function:
2 s^2 + 5 s + 1
-------------------
 s^2 + 2 s + 3
```

```
>> lsim(H,u,t)
```

Exercise 1:

Consider the transfer function

$$G(s) = \frac{6s\ \ + 1}{s^3 + 3s\ \ + 3s + 7}$$

Using MATLAB plot the pole zero map of the above system

Exercise 2:

a. Obtain the unit impulse response for the following system

$$\frac{B(s)}{A(s)} = \frac{1}{s\ \ + 0.2s + 1}$$

b. Obtain the unit step response for the following system

$$\frac{B(s)}{A(s)} = \frac{s}{s\ \ + 0.2s + 1}$$

c. Explain why the results in a. and b. are same?

Exercise 3:

A system has a transfer function

$$\frac{X(s)}{R(s)} = \frac{(15/z)(s + z)}{s\ \ + 3s + 15}$$

Plot the response of the system when R(s) is a unit impulse and unit step for the parameter z=3, 6 and 12.

Exercise 4:

Consider the differential equation $\ddot{y} + 4\dot{y} + 4y = u$ where $y(0) = \dot{y}(0) = 0$ and $u(t)$ is a unit step. Determine the solution analytically and verify by co-plotting the analytical solution and the step response obtained with ‘step’ function.

Laboratory Manual

for

Control System Lab

Prepared by

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Software Experiment 2: Block Diagram Reduction

Objective: The objective of this exercise will be to learn commands in MATLAB that would be used to reduce linear systems block diagram using series, parallel and feedback configuration.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

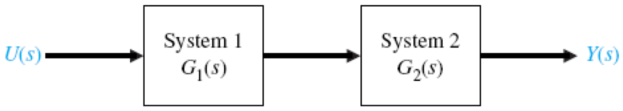
Category Soft-Experiment

Deliverables

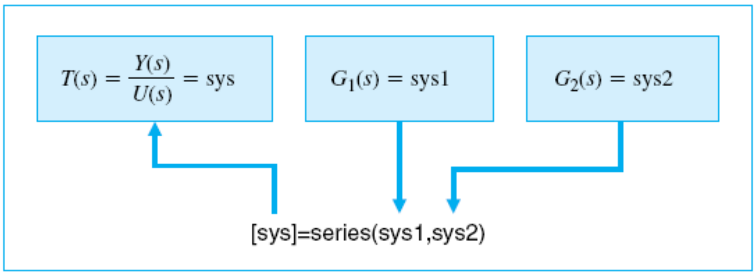
A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results for examples, exercises and Dorf (text book) related material of this lab should be reported properly.

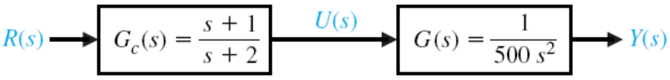
Series configuration: If the two blocks are connected as shown below then the blocks are said to be in series. It would like multiplying two transfer functions. The MATLAB command for the such configuration is “series”.



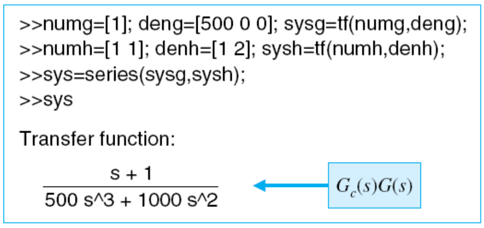
The series command is implemented as shown below:



Example 1: Given the transfer functions of individual blocks generate the system transfer function of the block combinations.

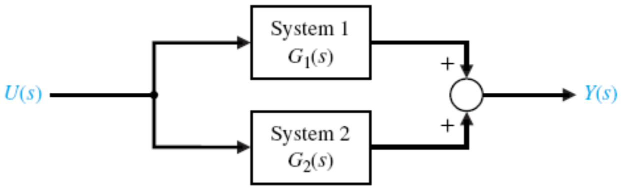


The result is as shown below:

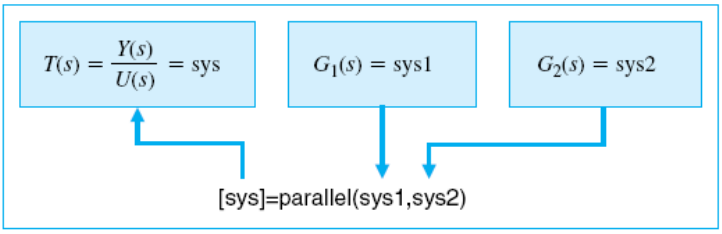


```
>>numg=[1]; deng=[500 0 0]; sysg=tf(numg,deng);
>>numh=[1 1]; denh=[1 2]; sysh=tf(numh,denh);
>>sys=series(sysg,sysh);
>>sys

Transfer function:
        s + 1
------------------
500 s^3 + 1000 s^2
```

Parallel configuration: If the two blocks are connected as shown below then the blocks are said to be in parallel. It would like adding two transfer functions.



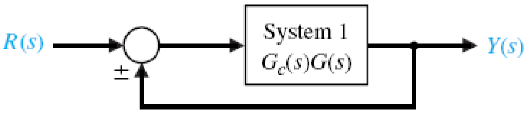
The MATLAB command for implementing a parallel configuration is “parallel” as shown below:



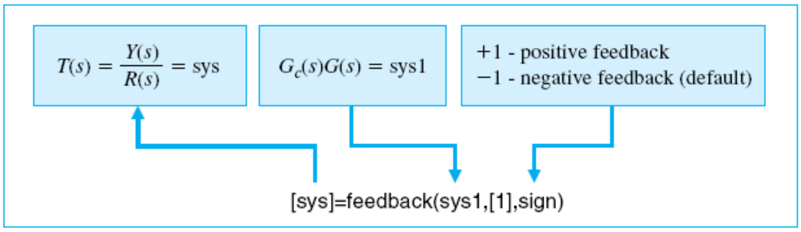
```
[sys]=parallel(sys1,sys2)
```

Example 2: For the previous systems defined, modify the MATLAB commands to obtain the overall transfer function when the two blocks are in parallel.

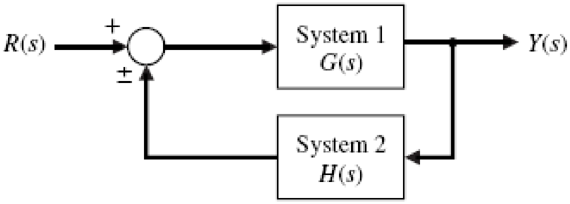
Feedback configuration: If the blocks are connected as shown below then the blocks are said to be in feedback. Notice that in the feedback there is no transfer function H(s) defined. When not specified, H(s) is unity. Such a system is said to be a unity feedback system.



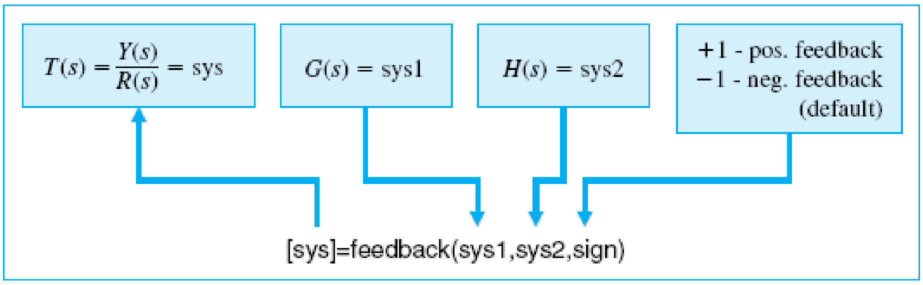
The MATLAB command for implementing a feedback system is “feedback” as shown below:



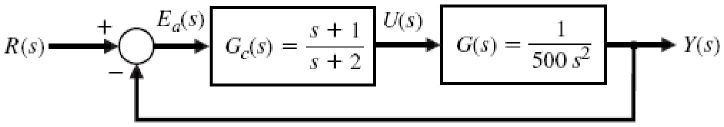
When H(s) is non-unity or specified, such a system is said to be a non-unity feedback system as shown below:



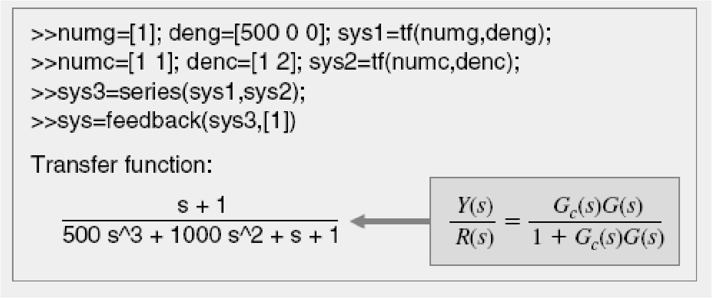
A non-unity feedback system is implemented in MATLAB using the same “feedback” command as shown:



Example 3: Given a unity feedback system as shown in the figure, obtain the overall transfer function using MATLAB:



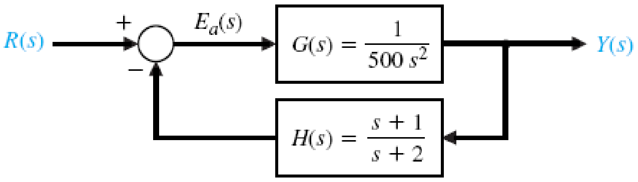
The result is as shown below:



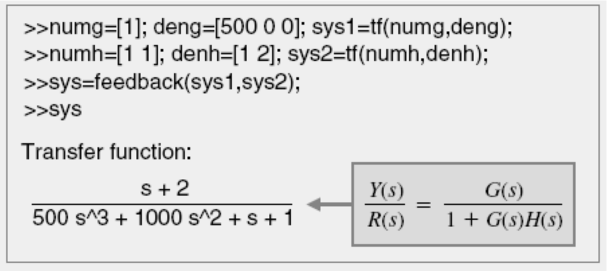
```
>>numg=[1]; deng=[500 0 0]; sys1=tf(numg,deng);
>>numc=[1 1]; denc=[1 2]; sys2=tf(numc,denc);
>>sys3=series(sys1,sys2);
>>sys=feedback(sys3,[1])

Transfer function:
            s + 1
  ------------------------
  500 s^3 + 1000 s^2 + s + 1
```

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}$$

Example 4: Given a non-unity feedback system as shown in the figure, obtain the overall transfer function using MATLAB:



The result is as shown below:

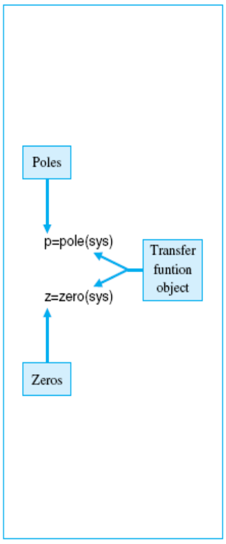


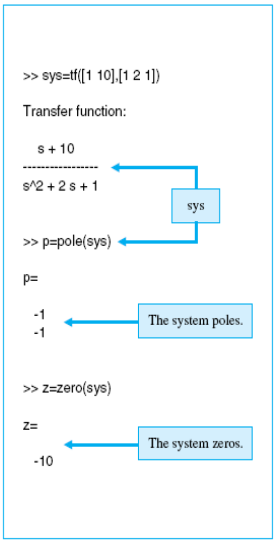
```
>>numg=[1]; deng=[500 0 0]; sys1=tf(numg,deng);
>>numh=[1 1]; denh=[1 2]; sys2=tf(numh,denh);
>>sys=feedback(sys1,sys2);
>>sys

Transfer function:
              s + 2
------------------------------
500 s^3 + 1000 s^2 + s + 1
```

Poles and Zeros of System: To obtain the poles and zeros of the system use the MATLAB command “pole” and “zero” respectively as shown in example 5. You can also use MATLAB command “pzmap” to obtain the same.

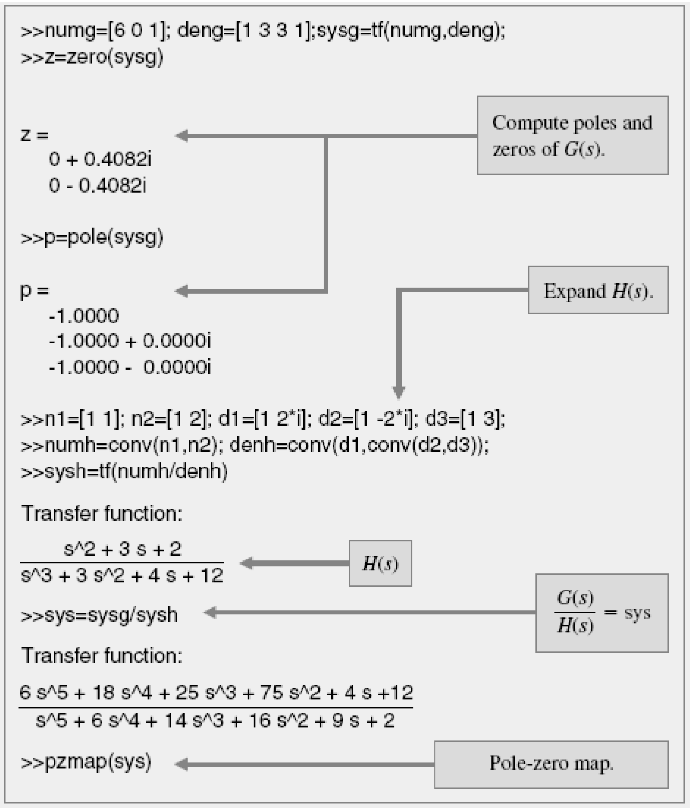
Example 5: Given a system transfer function plot the location of the system zeros and poles using the MATLAB pole-zero map command.

For example:



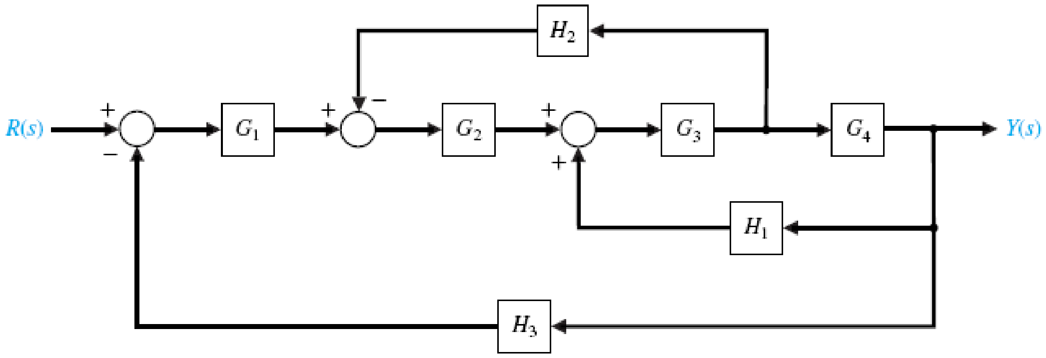


```
>> sys=tf([1 10],[1 2 1])

Transfer function:

  s + 10
-----------------
s^2 + 2 s + 1

>> p=pole(sys)

p=

  -1
  -1

>> z=zero(sys)

z=

  -10
```



```
>>numg=[6 0 1]; deng=[1 3 3 1];sysg=tf(numg,deng);
>>z=zero(sysg)

z =
    0 + 0.4082i
    0 - 0.4082i

>>p=pole(sysg)

p =
    -1.0000
    -1.0000 + 0.0000i
    -1.0000 -  0.0000i

>>n1=[1 1]; n2=[1 2]; d1=[1 2*i]; d2=[1 -2*i]; d3=[1 3];
>>numh=conv(n1,n2); denh=conv(d1,conv(d2,d3));
>>sysh=tf(numh/denh)

Transfer function:
      s^2 + 3 s + 2
-----------------------
s^3 + 3 s^2 + 4 s + 12

>>sys=sysg/sysh

Transfer function:
6 s^5 + 18 s^4 + 25 s^3 + 75 s^2 + 4 s +12
------------------------------------------
  s^5 + 6 s^4 + 14 s^3 + 16 s^2 + 9 s + 2

>>pzmap(sys)
```

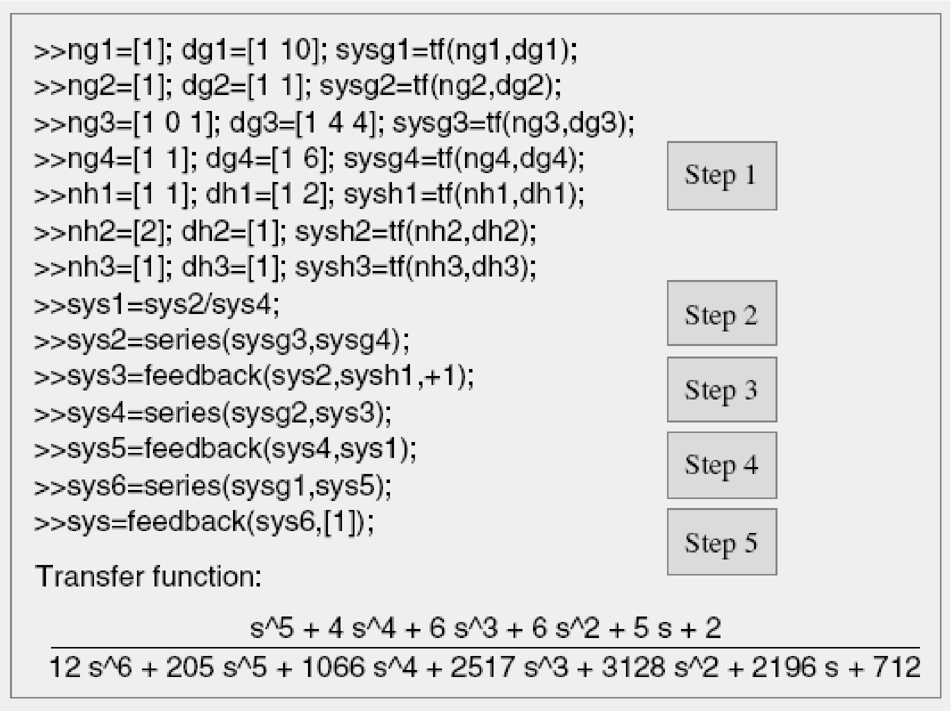
Exercise 1: For the following multi-loop feedback system, get closed loop transfer function and the corresponding pole-zero map of the system.



Given $G_1 = \dfrac{1}{(s+10)}$; $G_2 = \dfrac{1}{(s+1)}$; $G_3 = \dfrac{s^2+1}{(s^2+4s+4)}$; $G_4 = \dfrac{s+1}{(s+6)}$; $H_1 = \dfrac{s+1}{(s+2)}$; $H_2 = 2$

; $H_3 = 1$ (Reference: Page 113, Chapter 2, Text: Dorf.)

MATLAB solution:

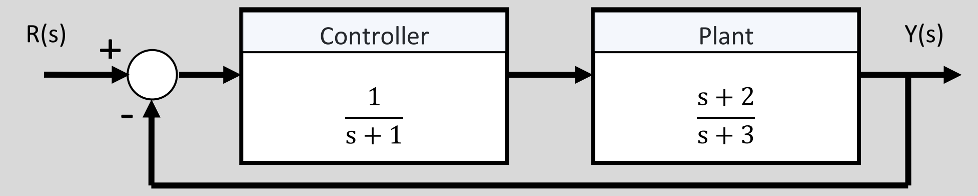


```
>>ng1=[1]; dg1=[1 10]; sysg1=tf(ng1,dg1);
>>ng2=[1]; dg2=[1 1]; sysg2=tf(ng2,dg2);
>>ng3=[1 0 1]; dg3=[1 4 4]; sysg3=tf(ng3,dg3);
>>ng4=[1 1]; dg4=[1 6]; sysg4=tf(ng4,dg4);
>>nh1=[1 1]; dh1=[1 2]; sysh1=tf(nh1,dh1);
>>nh2=[2]; dh2=[1]; sysh2=tf(nh2,dh2);
>>nh3=[1]; dh3=[1]; sysh3=tf(nh3,dh3);
>>sys1=sys2/sys4;
>>sys2=series(sysg3,sysg4);
>>sys3=feedback(sys2,sysh1,+1);
>>sys4=series(sysg2,sys3);
>>sys5=feedback(sys4,sys1);
>>sys6=series(sysg1,sys5);
>>sys=feedback(sys6,[1]);

Transfer function:

              s^5 + 4 s^4 + 6 s^3 + 6 s^2 + 5 s + 2
 -----------------------------------------------------------------
 12 s^6 + 205 s^5 + 1066 s^4 + 2517 s^3 + 3128 s^2 + 2196 s + 712
```

Instruction: Please refer to Section 2.6 and Section 2.2 in Text by Dorf.

Exercise 2: Consider the feedback system depicted in the figure below

a. Compute the closed-loop transfer function using the ‘series’ and ‘feedback’ functions
b. Obtain the closed-loop system unit step response with the ‘step’ function and verify that final value of the output is 2/5.



Reference: Please see Section 2.5 of Text by Dorf for Exercise 3.

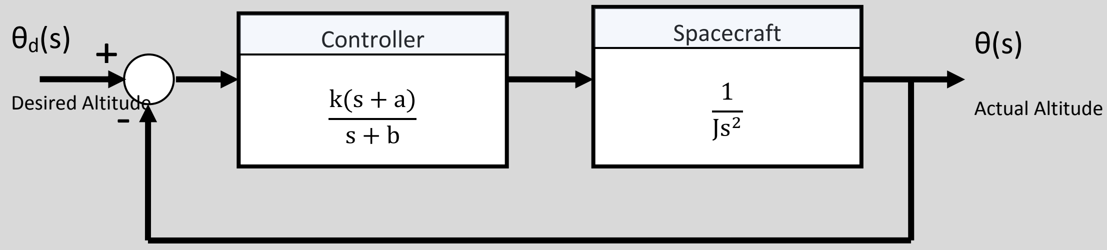
Exercise 3: A satellite single-axis altitude control system can be represented by the block diagram in the figure given. The variables ‘k’, ‘a’ and ‘b’ are controller parameters, and ‘J’ is the spacecraft moment of inertia. Suppose the nominal moment of inertia is ‘J’ = 10.8E8, and the controller parameters are k=10.8E8, a=1, and b=8.

a. Develop an m-file script to compute the closed-loop transfer function

$$T(s) = \theta(s)/\theta\ \ (s).$$

b. Compute and plot the step response to a 10^{o} step input.

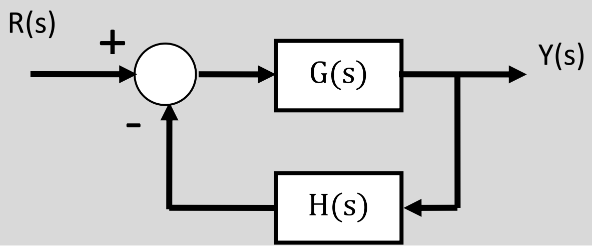
c. The exact moment of inertia is generally unknown and may change slowly with time. Compare the step response performance of the spacecraft when J is reduced by 20% and 50%. Discuss your results.



Reference: Please see Section 2.9 of Text by Dorf for Exercise 4.

Exercise 4: Consider the feedback control system given in figure, where

$$G(s) = \frac{s+1}{s+2} \text{ and } H(s) = \frac{1}{s+1}.$$



a. Using an m-file script, determine the close-loop transfer function.

b. Obtain the pole-zero map using the ‘pzmap’ function. Where are the closed-loop system poles and zeros?

c. Are there any pole-zero cancellations? If so, use the ‘minreal’ function to cancel common poles and zeros in the closed-loop transfer function.

d. Why is it important to cancel common poles and zeros in the transfer function?

Exercise 5: Do problem CP2.6 from your text

Laboratory Manual

for

Control System Lab

Prepared by

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Experiment: To study the performance of various types of controllers used to control the temperature of an oven.

Apparatus Required: Temperature control is one of the most common industrial control systems that are in operation. This equipment is designed to expose the students to the intricacies of such a system in the ‘friendly’ environment of a laboratory, free from disturbances and uncertainties of plant prevalent in an actual process. The ‘plant’ to be controlled is a specially designed oven having a short heating as well as cooling time. The temperature time data may be obtained manually, thus avoiding expensive equipment like an X-Y recorder or a pen recorder. A solid-state temperature sensor converts the absolute temperature information to a proportional electric signal. The reference and actual temperature are indicated in degree Celsius on a switch selectable digital display.

The Controller unit compares the reference and the measured signals to generate the error. Controller options available to the users consist of ON-OFF or relay with two hysteresis settings and combination of proportional, derivative and integral blocks having independent coefficient settings. A block diagram of the complete system is shown in Fig. 1.

Theory: The first step in the analysis of any control system is to derive its mathematical model. The various blocks shown in Fig. 1 are now studied in detail and their mathematical descriptions are developed. This would help in understanding the working of the complete system and also to implement control strategies.

The plant (oven): Plant to be controlled is an electric oven, the temperature of which must adjust itself in accordance with the reference of heat from one section to another. In the present case we are interested in the transfer of heat from the heater coil to the oven and leakage of heat from the oven to the atmosphere. Such systems may be conveniently analysed in terms of thermal resistance and capacitance, as explained below. However, this analysis is not very accurate, since the transfer of heat essentially takes pace form every part of the oven-thermal resistance and capacitance are obviously distributed. The lumped parameter model described here is therefore only an approximation. For a precise analysis, a distributed parameter model must be used. Another difficulty associated with temperature control systems is that whereas the temperature rise is produced by energy input, which is controllable, the temperature fall is due to heat loss which is uncontrollable and unpredictable. This implies that the oven will have different time constant while heating and cooling. Such a system is therefore rather difficult to control.

There are three modes of heat transfer viz. conduction, convection and radiation. Heat transfer through radiation maybe neglected in the present case since the temperature involved are quite small. For conductive and convective heat transfer

$$\theta = \alpha\Delta T$$

where, θ= rate of heat flow in Joule/sec.

ΔT= temperature difference in °C

α=constant

Under assumptions of linearity, the thermal resistance is defined as, R=Temperature-difference/rate of heat flow= $\Delta T/\theta = 1/\alpha$. This is analogous to electrical resistance defined by I=V/R. in a similar manner thermal capacitance of the mass is defined by

$$\theta = C\frac{d(\Delta T)}{dT}$$

Which is analogous to the V-I relationship of a capacitor, namely $I = C\frac{dV}{dt}$. In the case of heat,

$$C = Rate\ of\ heat\ flow/Rate\ of\ temperature\ change$$

The equation of an oven may now be written by combining the above two equation, implying that a part of the heat input is used in increasing the temperature of the oven and the rest goes out as loss. Thus

$$\theta = C\frac{dT}{dt} + \left(\frac{1}{R} \times T\right)$$

with the initial condition T(t = 0) $= T_{amb}$. Now, taking the Laplace transform with zero initial condition

$$\frac{T(s)}{\theta(s)} = \frac{R}{1 + sCR}$$

An analogous electrical network and block diagram may be drawn as in fig. 2 defined by the equation

$$I = C\frac{dV}{dt} + \frac{V}{R}$$

Eq. 1 is an extremely simplified representation of the thermal system under consideration and it gives rise to transfer function of the first order and type zero. Such a system should be easily controlled in the closed loop. Difficulties are however faced in the system due to the following reasons:

a) The temperature rise in response to the heat input is not instantaneous. A certain amount of time is needed to transfer the heat by convection and conduction inside the oven. This requires a delay or transportation lag term, exp(-sT_1), to be included in the transfer function, where T_1 is the time lag in seconds.
b) Unlike the equivalent electrical circuit of Fig. 2, the heat input in the thermal system cannot have a negative sign. This means that although the rate of temperature rise would depend

on the heat input, the rate of temperature fall would depend on thermal resistance R. The conventional analysis methods then become inapplicable.

c) Referring to the closed loop oven control system of fig. 3, it may be seen that in the steady state the error e_{ss} is given as

$$e_{ss} = \lim_{t\to\infty}\left(T_{ref} - T\right) \; = \; \frac{T_{ref}}{(1 + AR)}$$

In this system, A cannot be increased excessively in an attempt to reduce error, since a large gain is likely to lead to instability due to transportation lag. Also, every time $\left(T_{ref} - T\right)$ become negative, the heat input is cut off and the oven must cool down slowly. The temperature T therefore oscillates around the nominal value.

3.2 Controller

Basic control actions commonly used in temperature control systems are

- ON-OFF or relay
- Proportional
- Proportional-Integral
- Proportional-Integral-Derivative

These are described below in detail.

a) *ON-OFF or relay type controllers*, also referred to as two position controllers, consist of a simple and inexpensive switch/relay and are, therefore, used very commonly in both industrial and domestic control systems. Typical applications include air-conditioner and refrigerators, ovens, heaters with thermostat. Solenoid operated two position valves are commonly used in hydraulic and pneumatic systems. The basic input-output behavior of this controller is shown in Fig. 4. The two positions of the controller are M_1 and M_2, and H is the hysteresis or differential gap.

The hysteresis is necessary, as it enables the controller output to remain at its present value till the input or error has increased a little beyond zero. Hysteresis helps in avoiding too frequent switching of the controller, although a large value results in greater errors. The response of a system with ON-OFF controller is shown Fig. 5. Describing function technique is a standard method for the analysis of non-linear systems, for instance, one with an ON-OFF controller.

b) *Proportional controller* is simply an amplifier of gain K_p which amplifies the error signal and passes it to actuator. The noise, drift and bias currents of this amplifier set the lower limit of the input signal which may be handled reliably and therefore decide the minimum possible value of the error between the input signal and output. Also, the saturation characteristics of this amplifier sets the linear and non-linear regions of its operation. A typical proportional controller may have an input-output characteristic as in Fig. 6. Such controller gives non-zero steady state error to step input for a type-0 system as indicates earlier. The proportional (P) block in the system consists of a variable gain amplifier having a maximum value, K_{pmax} of 20.

c) *Proportional-Integral (PI) controller:* Mathematical equation of such a controller is given by

$$m(t) = K_p e(t) + K_I \int_0^t e(t)dt = K_p e(t) + \frac{1}{T_I}\int_0^t e(t)dt$$

and a block diagram representation is shown in fig. 7. It may be easily seen that this controller introduces a pole at the origin, i.e. increases the system type number by unity. The steady state error of the system is therefore reduced or eliminated. Qualitatively, any small error signal e(t), present in the system, would get continuously integrated and generate actuator signal m(t) forcing the plant output to exactly correspond to the reference input so that the error is zero. In practical systems, the error may not be exactly zero due to imperfections in an electronic integrator caused by bias current needed, noise and drift present and leakage of the integrator capacitor.

The integral (I) block in the present system is realised with the circuit shown in fig. 8 and has a transfer function

$$G_r(s) = \frac{1}{41s} = \frac{K_I}{s}$$

The integral gain is therefore adjustable in the range 0 to 0.024 (approx.). Due to the tolerance of large capacitance's, the value of K_I is approximate.

d) *Proportional-Integral-Derivative (PID) controller:* Mathematical equations governing the operation of this controller is as

$$\begin{aligned} m(t) &= K_p e(t) + K_I \int_0^t e(t)dt + K_D \frac{de(t)}{dt} \\ &= K_p e(t) + \frac{1}{T_I}\int_0^t e(t)dt \ + T_D \frac{de(t)}{dt} \end{aligned}$$

So that in the Laplace transform domain,

$$\frac{M(s)}{E(s)} = \left(K_p + T_D s + \frac{1}{T_I s}\right)$$

A simple analysis would show that the derivative block essentially increases the damping ratio of the system and therefore improves the dynamic performance by reducing overshoot. The PID controller therefore helps in reducing the steady state error with an improvement in the transient response.

The derivative (D) block in this system is realized with the circuit of fig. 9. This has a transfer function

$$G_D(s) = 19.97s\ (approx.)$$

The derivative gain is therefore adjustable in the range 0 to 20 approximately. Again, the approximation is due to the higher tolerance in the values of large capacitances.

PID controller is one of the most widely used controller because of its simplicity. By adjusting its coefficients K_p, K_D (or T_D) and K_I (or T_1) the controller can be used with a variety of systems. The process of setting the controller coefficients to suit a given plant is known as tuning. There are many methods of 'tuning' a PID controller. In the present experiment, the method of Ziegler-Nichol has been introduced which is suitable for the oven control system, although better methods are available and may be attempted.

Temperature Measurement

The oven temperature can be sensed by a verity of transducers like thermistor, thermocouple, RTD and IC temperature sensors. In the present setup, the maximum oven temperature is around 90°c which is well within the operating range of IC temperature sensor like AD590. Further, these sensors are linear and have a good sensitivity. The time constant of the sensor has however been neglected in the analysis since it is insignificant compared with the oven time constant.

Procedure

A variety of experiments may be conducted with the help of this unit. The principal advantage of the unit is that all power sources and metering are built-in and needs only a watch to be able to note down the temperature readings at precise time instants. After each run the oven must be cooled to nearly the room temperature, which may take about 15-20 minutes. This would limit the number of runs to about four in a usual laboratory class. The experiments suggested could be completed in about 6-8 hours.

Identification of oven parameters

Plant identification is the first step before an attempt can be made to control it. In the present case, the oven equations are obtained experimentally from its step response as outlined below:

In the open loop testing, the oven is driven through P- amplifier set to a gain of 10. The input to this amplifier is adjustable through reference potentiometer. This input can be seen on digital display, so that when you set 5.0°C, the input to proportional amplifier is 50mV and its output is 0.5V.

- Keep switch S_1 to 'WAIT', S_2 to 'SET' and open 'FEEDBACK' terminals.
- Connect P output to the driver input and switch ON the unit.
- Set P potentiometer to 0.5 which gives K_P=10. Adjust reference potentiometer to read 5.0 on the DVM. This provides an input of 0.5V to the driver.
- Put switch S_2 to the 'MEASURE' position and note temperature readings every 15 sec, till the temperature becomes almost constant.
- Plot the temperature-time curve on a graph paper. Calculate T_1 and T_2 and hence write the transfer function of the oven including its driver.

ON-OFF controller

- Keep switch S_1 to 'WAIT' position and allow the oven to cool to room temperature. Short 'FEEDBACK' terminals.
- Keep switch S_2 to the 'SET' position and adjust reference potentiometer to the desired output temperature, say 60.0°C, by seeing on the digital display.
- Connect R output to the driver input. Outputs of P, D and I must be disconnected from driver input. Select 'HI' or 'LO' value of hysteresis. (First keep the hysteresis switch to 'LO')
- Switch S_2 to 'MEASURE' and S_1 to 'RUN' position. Read and record oven temperature every 15/30 sec., for about 20 minutes.
- Plot a graph between temperature and time and observe the oscillations in the steady state. Note down the magnitude of oscillations.
- Repeat above steps with the 'HI' setting for hysteresis and observe the rise time, steady - state error and percent overshoot.

Proportional controller

Ziegler and Nichols suggest the value of K_P for P-Controller as:

$$K_P = \left(\frac{1}{K}\right) \times \frac{T_1}{T_2}$$

- Starting with a cool oven, keep switch S_1 to 'WAIT' position and connect P output to the driver input. Keep R, D and I outputs disconnected. Short 'FEEDBACK' terminals.
- Set P potentiometer to the above calculated value of K_P, keeping in mind that the maximum gain is 10.
- Plot the observations on a linear graph paper and observe the rise time, steady-state error and percent overshoot.

Proportional-Integral controller

Ziegler and Nichols suggested the value of K_P and K_1 for P-I controller as

$$K_P = \left(\frac{0.9}{K}\right) \times \frac{T_1}{T_2};\ T_1 = \frac{1}{K_1} = 3.3T_2;\ giving\ K_1 = \frac{1}{3.3T_2}$$

- Starting with a cool oven, keep switch S_1 to 'WAIT' connect P and I outputs to driver input and disconnected R and D outputs. Short feedback terminals.
- Set P and I potentiometers to the above values of K_P and K_1 respectively, keeping in mind that the maximum value of K_P is 20 and that of K_1 is 0.024.
- Select and set the desired temperature to say 60.0°C.
- Keep switch S_1 to 'RUN' position and record temperature readings as before.
- Plot the response on a graph paper and observe the steady state error and percent overshoot.

Proportional-Integral-Derivative controller

Ziegler and Nichols suggested the value of K_P and K_D and K_1 for tis controller as

$$K_P = \left(\frac{0.9}{K}\right) \times \frac{T_1}{T_2};\ T_1 = \frac{1}{K_1} = 3.3T_2;\ giving\ K_1 = \frac{1}{3.3T_2}$$

- Starting with a cool oven, keep switch S_1 to ‘WAIT’ connect P, D and I outputs to driver input. Keep R output disconnected. Short feedback terminals.
- Set P, I and D potentiometers according to the above calculated values of K_P, K_1 K_D keeping in mind that the maximum value for these are 20, K_P is 20, 0.024 and 23.5 respectively.
- Select and set the desired temperature, say 60.0°C.
- Switch S_1 to ‘RUN’ and record temperature readings.
- Plot the response on a linear graph paper and observe the rise time, steady state error and percent overshoot.
- Compare the results of the various controller options.

Further experimentation

The controller settings suggested by Ziegler and Nichols are not optimum. It is therefore possible to experiment with other methods available in the literature or to attempt trial and error settings. Students at the master’s level may attempt to calculate theoretically the optimum values of K_P , K_D and K_1 based on some performance criterion and then verify the results on the setup. It may be convenient to use a pen recorder or X-Y recorder for such experiments. A terminal has been provided at the back of the unit for this purpose with a sensitivity of 10mV/°C.

Additional laboratory work may involve modification of the oven parameters and then repeating the basic experiments. This may be done simply by putting thermal load into the oven, thus increasing its thermal capacitance or by providing insulation to the oven thus increasing its thermal resistance. These may also act as disturbance inputs to the oven while it is operating under steady-state conditions, and their effect may be studied.

Results

- **a) Open loop measurement:**
- **b) Calculation of K_P, K_I, K_D:**
  1. ***P Control*: K_P**
  2. ***PI Control:* K_I**
  3. ***PID Control:* K_D**
- **c) Results:**

Typical Readings:

Time (secs)	Temp. (degree c) **Set point = 30.0** room temp=23.0	Time (secs)	Temp. (degree c) **Set point = 30.0** room temp=23.0
0			23
15			25.2
30			26.6
45			28.1
60			29.1
75			29.6
90			29.9
105			30.0
120			30.0
135			29.9
150			29.8
165			29.7
180			29.5
195			29.3
210			29.2
225			29.0
240			28.8
255			28.7
270			28.7
285			28.6
300			28.6
315			28.6
330			28.6
345			28.6
360			28.6
375			28.6
390			28.6
405			28.6
420			28.6
435			28.6
450			28.6

Experiment: using PI controller	
Time (secs)	Temp. (degree c) **Set point = 30.0** room temp=23.0
0	23.5
15	24.9
30	26.5
45	28.6
60	29.8
75	30.4
90	30.6
105	31.0
120	31.1

135	31.0
150	31.0
165	30.8
180	30.7
195	30.5
210	30.3
225	30.1
240	29.9
255	29.8
270	29.6
285	29.4
300	29.3
315	29.2
330	29.1
345	29.0
360	29.0
375	29.0
390	29.0
405	29.0
420	29.0
435	29.0
450	29.0

Experiment: using PID controller	
Time (secs)	Temp. (degree c) **Set point = 30.0** room temp=23.0
0	23.5
15	24.8
30	26.6
45	28.4
60	29.7
75	30.3
90	30.8
105	31.0
120	31.1
135	31.1
150	31.0
165	30.9
180	30.8
195	30.6
210	30.5
225	30.3
240	30.1
255	30.0
270	29.6
285	29.6

300	29.5
315	29.3
330	29.2
345	29.1
360	29.1
375	29.1
390	29.1
405	29.1
420	29.1
435	29.1
450	29.1

Experiment: using Relay control	
Time (secs)	Temp. (degree c) **Set point = 30.0** Room temp.=23.0
0	23.6
15	25.0
30	28.4
45	32.0
60	34.1
75	35.2
90	35.6
105	36.0
120	35.9
135	35.8
150	35.5
165	35.2
180	34.9
195	34.5
210	34.0
225	33.7
240	33.3
255	32.9
270	32.5
285	32.1
300	31.8
315	31.4
330	31.1
345	30.8
360	30.5
375	30.3
390	30.0
405	30.0
420	30.0
435	30.0
450	30.0

Experiment: using P controller (open loop)	
Time (secs)	Temp. (degree c) **Set point = 30.0** room temp=23.0
0	23.7
15	24.6
30	27.3
45	31.1
60	35.5
75	40.2
90	45.0
105	49.8
120	54.4
135	58.7
150	62.8
165	66.5
180	70.0
195	73.3
210	76.2
225	79.0
240	81.5
255	83.8
270	86.0
285	87.9
300	89.6
315	91.2
330	92.7
345	94.0
360	95.2
375	96.2
390	97.2
405	98.2
420	99.0
435	--
450	--